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OM

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ACE

ECE

PM 1 (B)

Communication System

☆ Digital Communication

- **Baseband data Tx**
  - **Analog**
    - PAM
    - PWM
    - PPM
  - **Digital**
    - PCM
    - DPCM
    - DM
    - ADM
- **Bandpass data Tx**
  - **Binary signalling**
    - ASK
    - FSK
    - PSK (or) BPSK
  - **M-ary signalling**
    - QPSK
    - 8SK
    - 16SK

* Review of Sampling Theorem:

⇒ In digital communications binary data is transmitted through the channel. To convert an analog signal into a digital signal, The signal should be sampled for every T_s seconds. This samples are applied to an Analog to Digital Converter

to generate the binary data. At the receiver DAC is convert the binary into samples. Finally a LPF is used to reconstruct the signal from samples.

But the signal reconstruction is possible only when the following condition is satisfied.

$$\boxed{\frac{1}{T_s} \geq 2 f_m \quad \text{sample/sec.}}$$

⇒ The minimum sampling rate require to reconstruct the signal is called as the Nyquist rate,

$$\therefore \boxed{\frac{1}{T_s} = 2 f_m \quad \text{sample/sec.}} \leftarrow \text{H.B}$$

⇒ If the sampling rate is greater than the Nyquist rate then the signal is over sampled.

$$\boxed{\frac{1}{T_s} > 2 f_m.}$$

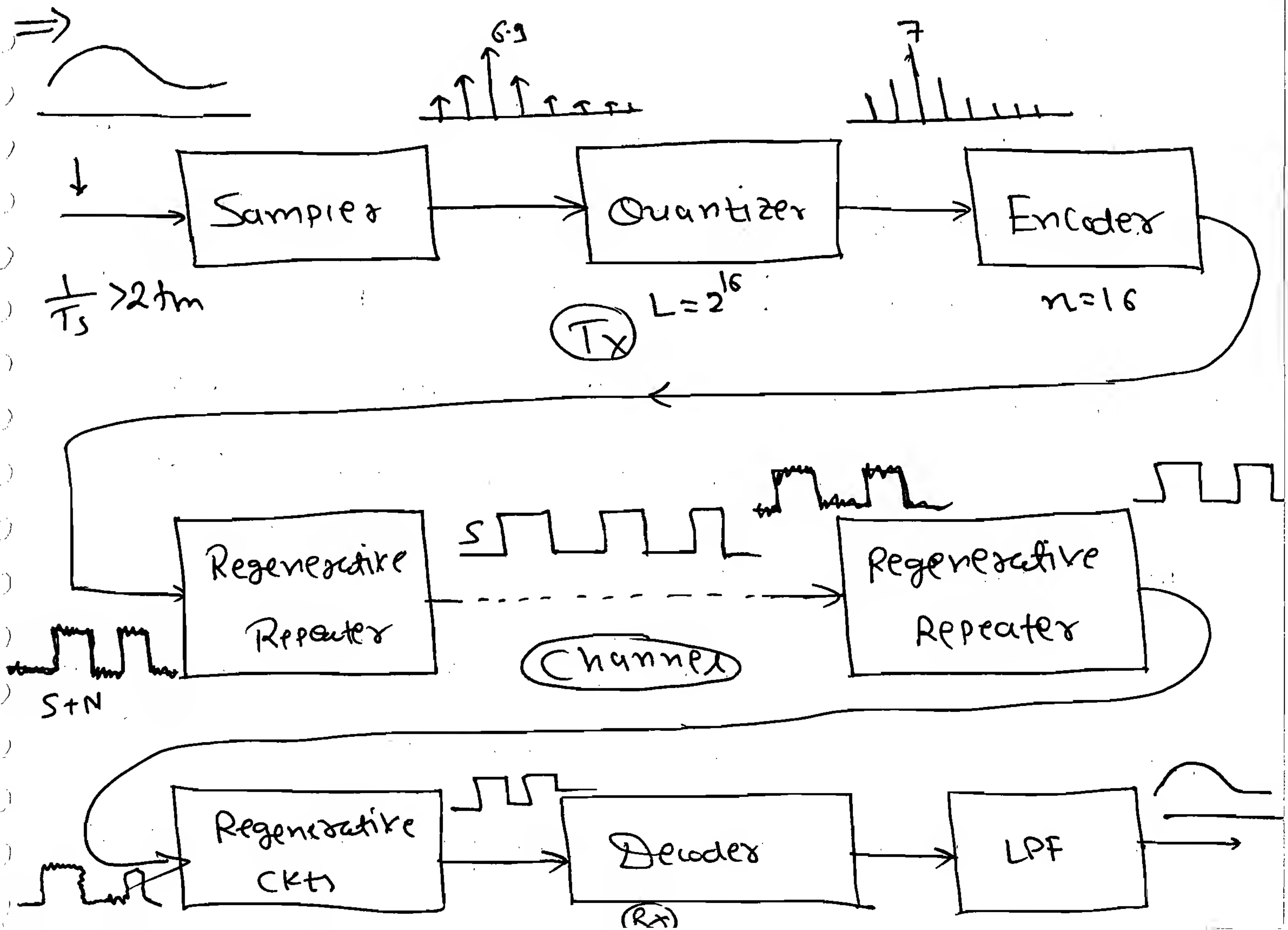
⇒ If the Sampling rate is less than the Nyquist rate then the signal is under sampled and distortion will occured.

$$\boxed{\frac{1}{T_s} < 2f_m} \quad \times$$

⇒ The o/p of the LPF is envelope of the sampled signal.

(1) Pulse Code Modulation (PCM):-

⇒



⇒ Sampler Converts the continuous time signal into the descrete time signal but the signal should be over sampled ($\frac{1}{T_s} > 2f_m$).

⇒ In Quantizer each sample is rounded off to the nearest quantization level.

⇒ In Voice transmission in telephone system, the sampling rate used is 8000 sample/sec and each sample is encoded into 8 bits.

⇒ In Audio CD Recording, the Sampling rate used is 44,100 sample/sec and each sample is encoded into 16 bits.

⇒ Encoder o/p is the Binary data which is represented in the form of Rectangular Pulses.

⇒ When the binary data is transmitted through a channel, amplitude distortion occurs due to noise. Regenerative repeaters

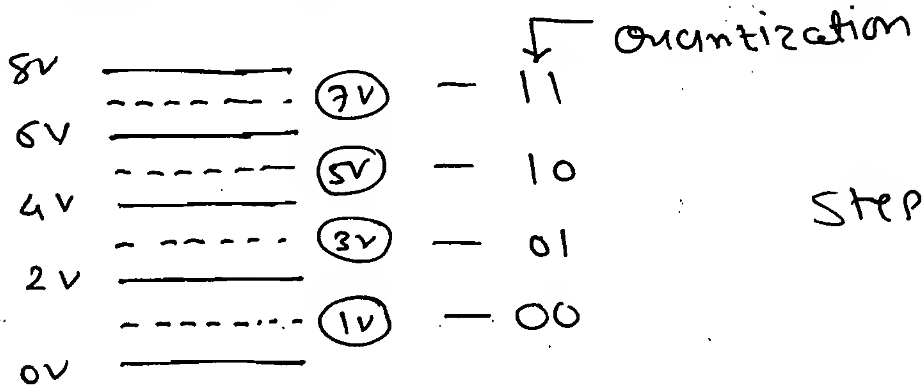
⇒ Regenerative Repeaters are used to eliminate noise from the signal and the noise present at the input of the receiver.

⇒ The decoder is a analog converter which convert the binary data into sample.

⇒ The LPF is used to reconstruct the signal from the samples.

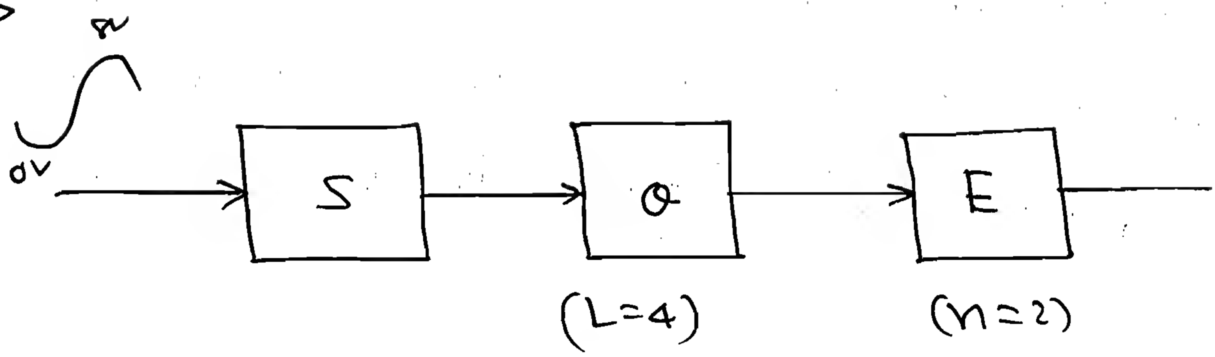
* Quantizer.

⇒ Consider the 2 bit PCM System and assume that amplitude varies from 0 to 8. The max. Sampled value applied to the Quantizer is 8V and the min. value is 0V. This entire range is divided into 4 equal step as shown in fig.



Step size = 2V.

⇒



S	Q	E
1.3	1	00
5.9	5	10
8	7	11

⇒ Consider an n-bit PCM System

where

→ n = no. of bits per Sample.

→ L = no. of Quantization level

$\boxed{L = 2^n}$ ← I.B

→ $\boxed{\Delta = \text{Step size} = \frac{V_{max} - V_{min}}{L}}$ ← I.B

$\Delta = \frac{8-0}{4} = 2V$

→ $\boxed{\Delta = \frac{V_{pp}}{L}}$ ← I.B.

→ Q_e = Quantization error.

$\boxed{Q_e = \text{Sampled value} - \text{Quantized value}}$

↑ I.B

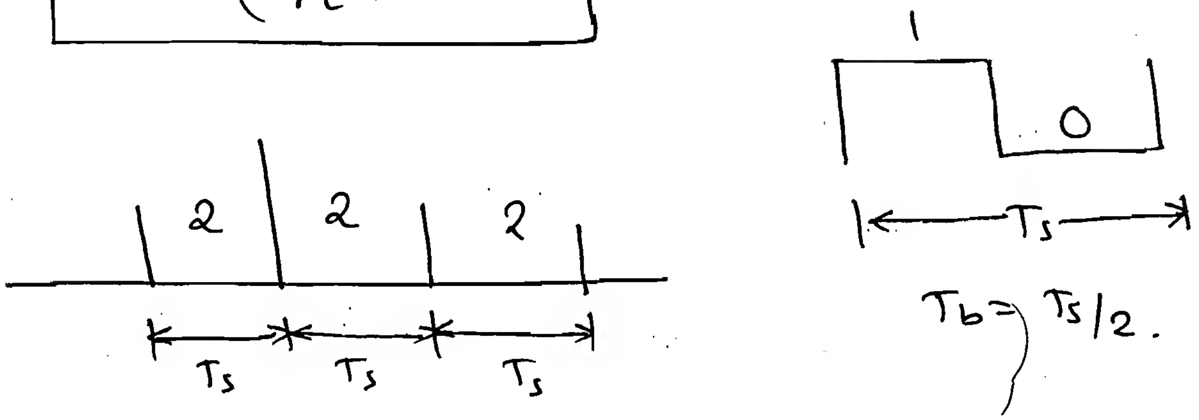
$\therefore Q_e = 1.3 - 1 = 0.3\ V.$

$\Rightarrow$ $\boxed{[Q_e]_{max} = \frac{\Delta}{2}}$ [Q_e varies from $-\frac{\Delta}{2}$ to $+\frac{\Delta}{2}$].

$\uparrow$ H.B.

$\Rightarrow$ T_b = bit duration

$\boxed{T_b = \left(\frac{T_s}{n}\right) \text{ sec.}}$ $\leftarrow$ H.B.



$T_b = T_s/2$.

$\Rightarrow$ R_b = Bit rate = bits/sec.

H.B.

$\boxed{R_b = [\text{Sampling rate} \times n] \text{ bits/sec.}}$ — ①

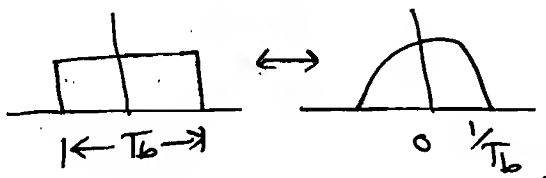
$= \frac{\text{sample}}{\text{sec}} \times \frac{\text{bits}}{\text{samp.}} = \text{bits/sec.}$

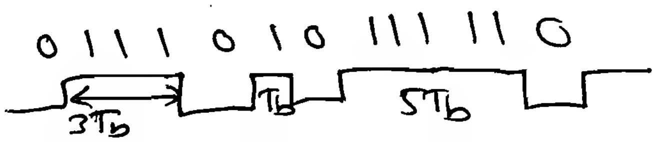
$\rightarrow$ $R_b = \frac{1}{T_s} \times n = \frac{n}{T_s}$ bps.

H.B. $\rightarrow$ $\boxed{R_b = \frac{1}{T_b} \text{ bps.}}$ — ②

$\Rightarrow$ $\boxed{(BW)_{max} = \left(\frac{1}{T_b}\right) \text{Hz.}}$

H.B.





Q-1 A signal whose amplitude varies from 0 to 10 V is Band limited to 4 KHz and transmitted through a channel using 5 bit PCM. The sampling rate is 50% higher than the Nyquist rate. Calculate all parameters of a PCM system.

Sol^n:

$f_m = 4 \text{ KHZ}$, $V_{max} = 10V$, $V_{min} = 0V$.

$n = 5$ bit

Sampling rate = 1.5 of Nyquist rate

$= 1.5 \times (2 f_m)$

$= \frac{3}{2} \times 2 \times 4000$

Sampling rate $= 12000$ sample / sec $= \frac{1}{T_s}$

→ $L = 2^n = 2^5 = 32$ = Quantization level.

$\Delta = \frac{V_{max} - V_{min}}{L} = \frac{10-0}{32} = \frac{10}{32}$ V.

→ $[Q_e]_{max} = \frac{\Delta}{2} = \frac{10}{64}$ Volts.

T_b = bit duration $= \left(\frac{T_s}{n}\right)$ sec

$T_b = \frac{1}{12000 \times 5}$

$T_b = \frac{1}{60000}$ sec.

$\therefore \quad R_b = 12000 \times 5$

$R_b = 60000$ bits/sec

$\boxed{R_b = 60 \text{ KbPS.}}$

$$(BW)_{max} = \frac{1}{T_b} = 60 \text{ KHZ.}$$

→ The minimum BW of the channel to transmitt the PCM signal without any distortion is 60 KHZ.

$\boxed{Q-2}$ A vedio signal is Bandlimited to 4.5 MHZ and Transmitted through a channel using PCM.

① Determine the Samping rate if the signal is sampled at a rate of 20% higher than the nyquist rate.

② Determine the bit rate if the no. of quantization level is 1024.

Soln:

$f_m = 4.5$ MHZ

⇒ Nyquist rate $= 2 f_m = 9 \times 10^6$ sample / sec.

→ Samping rate $= 1.2 \times$ Nyquist rate

Samping rate $= 10.8 \times 10^6$ sample/rate.

② $L = 1024$

$\therefore L = 2^n \Rightarrow \boxed{n=10}$

bit rate $R_b = \left(\frac{1}{T_s} \times n\right)$

$\therefore R_b = 10.8 \times 10^6 \times 10$

$\therefore R_b = 10.8 \times 10^7$.

$\boxed{R_b = 108 \text{ MHz/sec.}}$

$(BW)_{max} = 108$ MHZ.

IMP

Q-3 A sinusoidal signal is Band limited to 5 KHZ and transmitted through a channel using PCM. The sampling rate is twice the nyquist rate. The max. quantization error should be 0.1 % of the peak signal amp. Determine the bit rate.

Soln:

$f_m = 5$ KHZ

$\Rightarrow$ Nyquist rate $= 2f_m = 10{,}000$ sample/sec.

$\Rightarrow$ Sampling rate $= 2$ (N.R.).

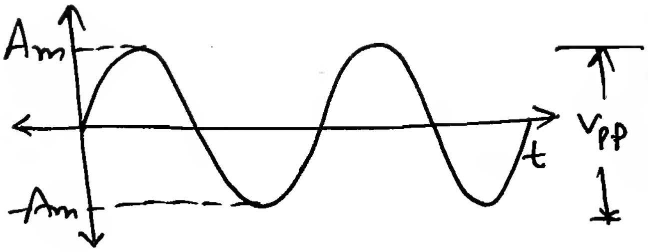
$= 20{,}000$ sample/sec.

$[Q_e]_{max} = \frac{\Delta}{2} = \frac{V_{P-P}}{2L}$.

$\Rightarrow$ Peak-value $V_P = A_m$

Peak-peak value

$V_{P-P} = 2A_m.$



$$[Q_e]_{max} = \frac{0.1}{100} \times V_P.$$

$$\therefore \frac{V_{P-P}}{2L} = \frac{0.1}{100} \times V_P.$$

$$\therefore \frac{\cancel{2}\cancel{A_m}}{\cancel{2}L} = \frac{0.1}{100} \times \cancel{A_m}$$

$$\therefore \boxed{L = 1000}$$

$$\Rightarrow 2^n = L = 1000 \Rightarrow \boxed{n = 10}$$

$$R_b = \left(\frac{1}{T_s} \times n\right) \text{ bps}$$

$$\therefore R_b = 20{,}000 \times 10.$$

$$\boxed{R_b = 0.2 \text{ Mbps.}} \Rightarrow \boxed{B_{max} = 0.2 \text{ MHz}}$$

(Q-4) A signal $m(t) = 4\cos 10^3 t \times 2\pi$ is sampled at Nyquist rate and transmitted through a channel using 3 bit PCM.

① Calculate all parameters.

② If the sampled values are 3.8, 2.8, 2.1, 1.7, −0.5, −3.2, −4.

Determine the quantization o/p and quantization error for each sample.

③ Sketch the transfer char. of quantizer.

Solⁿ: 5

① $V_p = A_m = 4$ V.

$V_{p\text{-}p} = 2 \times V_p = 8$ V.

$V_{max} = 4$ V, $V_{min} = -4$ V.

→ $n = 3$ bit

$L = 2^n = 2^3 = 8$ level.

$$\Delta = \frac{V_{p\text{-}p}}{L} = \frac{8}{8} = 1 \text{ V} = \text{Step size.}$$

$$[Q_e]_{max} = \frac{\Delta}{2} = \frac{1}{2} = 0.5 \text{ V.}$$

→ $f_m = 1$ KHz

Samping rate = Nyquist rate = $2 f_m$.

∴ Samping rate $= \frac{1}{T_s} = 2000$ sample/sec.

→ Bit rate $R_b = \frac{1}{T_s} \times n$

$\Rightarrow R_b = 2000 \times 3 = 6000$ bps

$R_b = 6$ KbPS.

$$(BW)_{max} = \frac{1}{T_b} = 6 \text{ KHZ.}$$

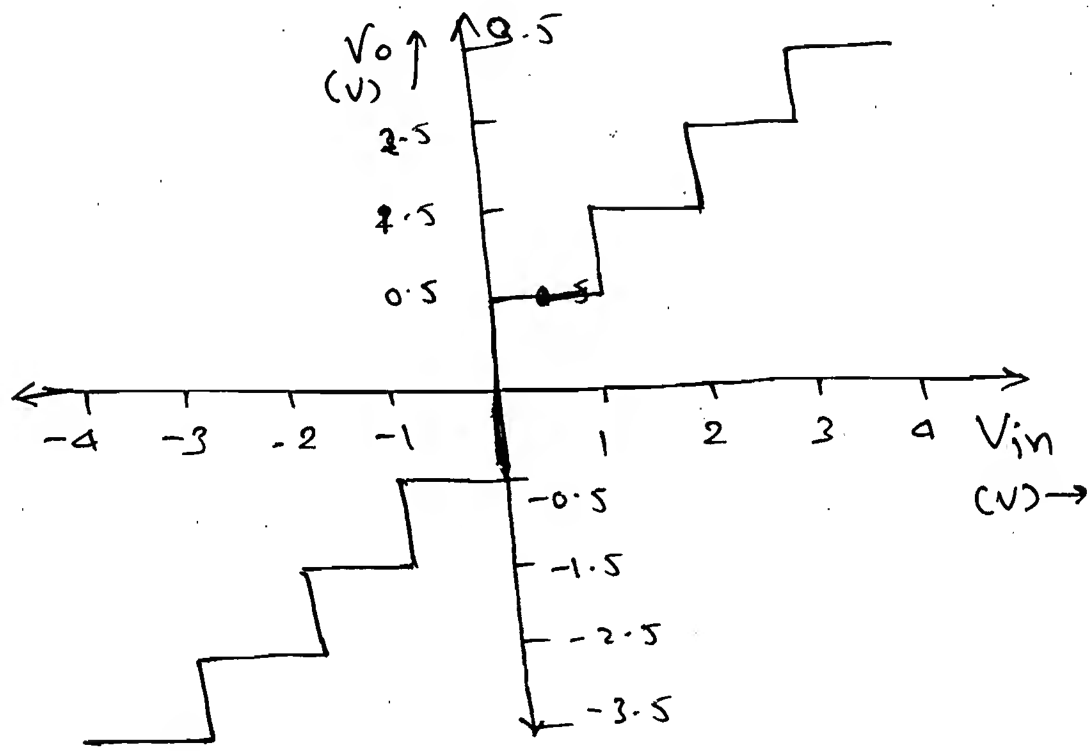
→ Bit duration $= T_b = \frac{T_s}{n} = \frac{1}{6000}$ sec.

② Sample values are 3.8, 2.8, 2.1, 1.7,
−0.5, −3.2, −4.

S(V)	Q(V)	Q_e(V) $=S-Q$	E
3·8	3·5	0·3	111
2·1	2·5	−0·4	110
1·7	1·5	0·2	101
−0·5	−0·5	0	011
−3·2	−3·5	0·3	000
−4	−3·5	−0·5	000

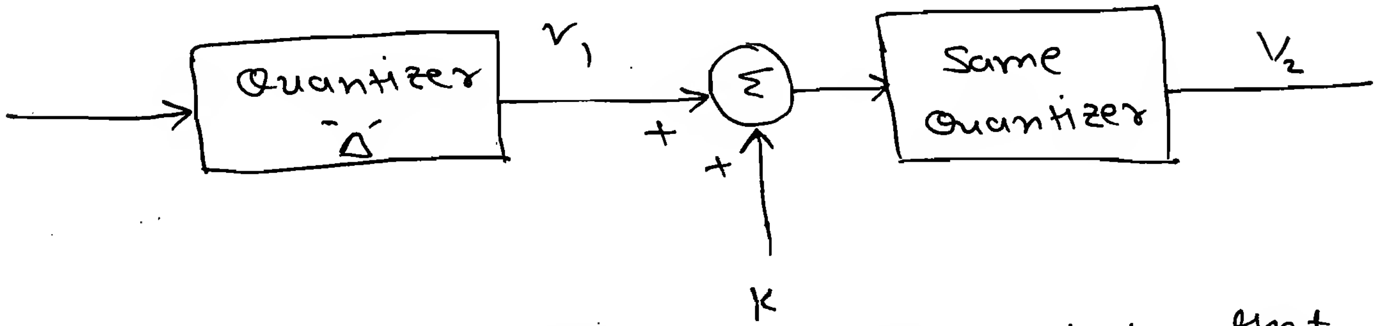
Q-level	Encoder O/P.
−3·5 →	000
−2·5 →	001
−1·5 →	010
−0·5 →	011
+0·5 →	100
+1·5 →	101
+2·5 →	110
+3·5 →	111

③ Transfer Chara.



Q-5 Consider a system as shown in fig.

Quantizer stepsize = Δ



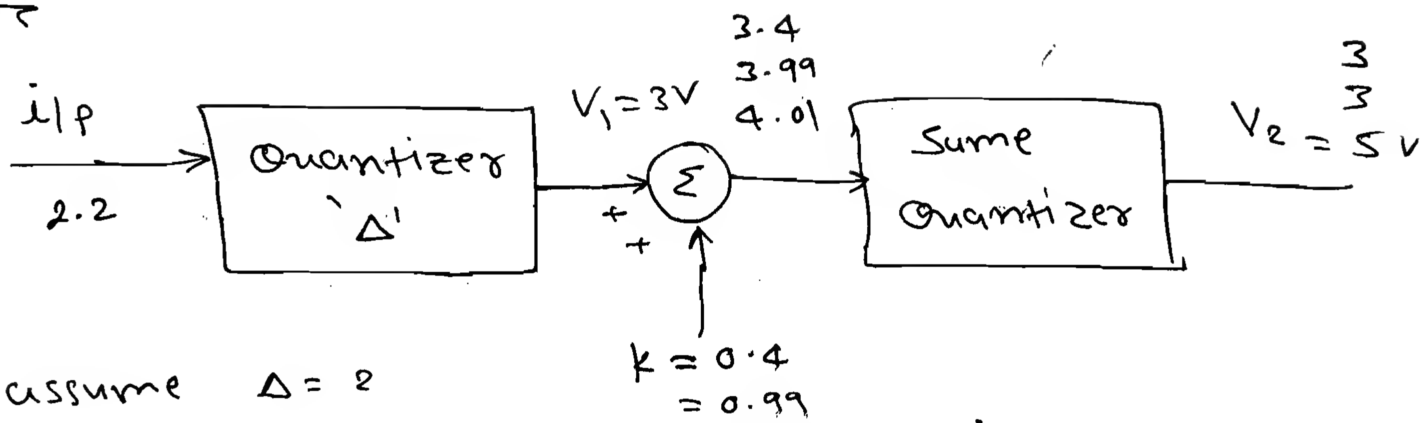
Determine the minimum value of K that should be added to v_1 so that the V_1 &

V_2 are different.

(A) Δ (B) 2Δ

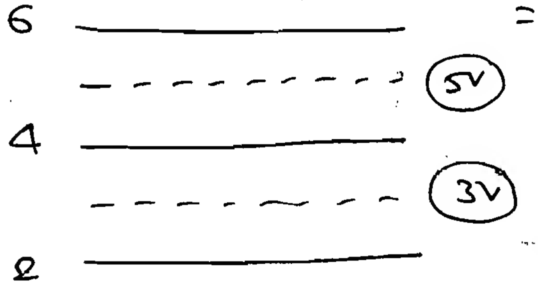
(C) $\frac{\Delta}{2}$ (D) Δ^2.

Sol^n:



let, i/p = 2.2 V

assume $\Delta = 2$



$k = 1.01 = \frac{\Delta}{2}$.

So, Ans is $\frac{\Delta}{2}$.

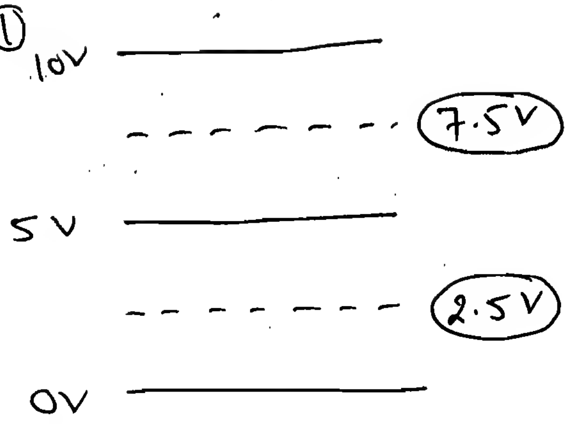
$\boxed{Q-6}$ The i/p to the Quantizer Varies from 0 to 10 volts.

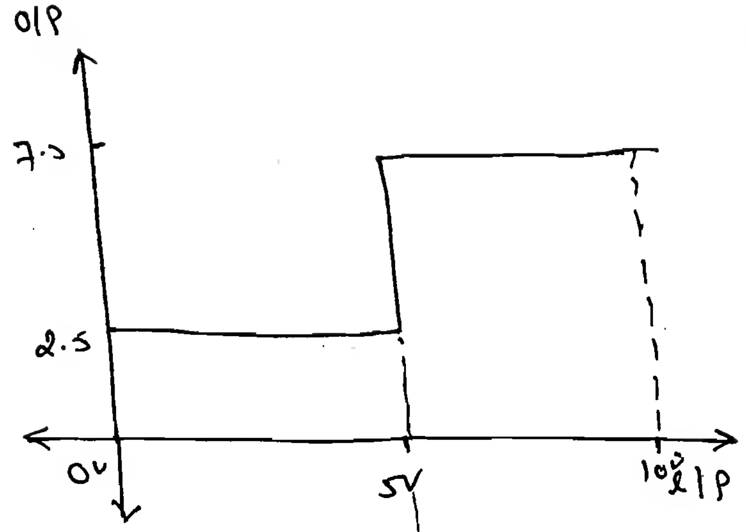
(1) When the input lies betn 0 to 5V the o/p is 2.5 V. and from 5 to 10 o/p lies is 7.5V Sketch the transfer chara. and Variation of Quantization error.

(2) When the i/p lies betn 0 to 5V. The o/p is 2V and when the i/p betn 5 to 10V the o/p is 9V. Sketch the transfer chara. and Variation of Quantization error.

Soln:

1





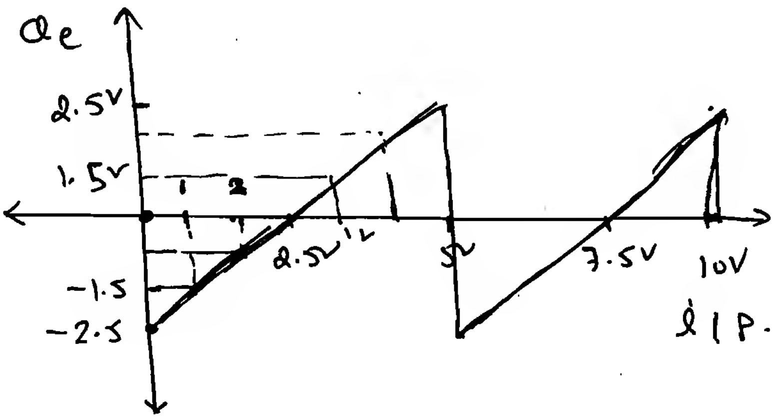
Transfer chara.

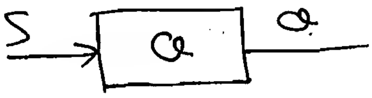
Step size 'Δ' = 5V

$[Q_e]_{max} = \frac{\Delta}{2} = 2.5V$

Q_e Varies betn

$\frac{\Delta}{2}$ to $\frac{-\Delta}{2}$ i.e. 2.5V to -2.5V

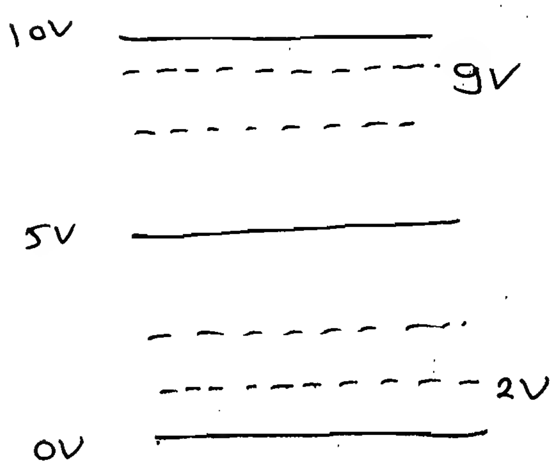


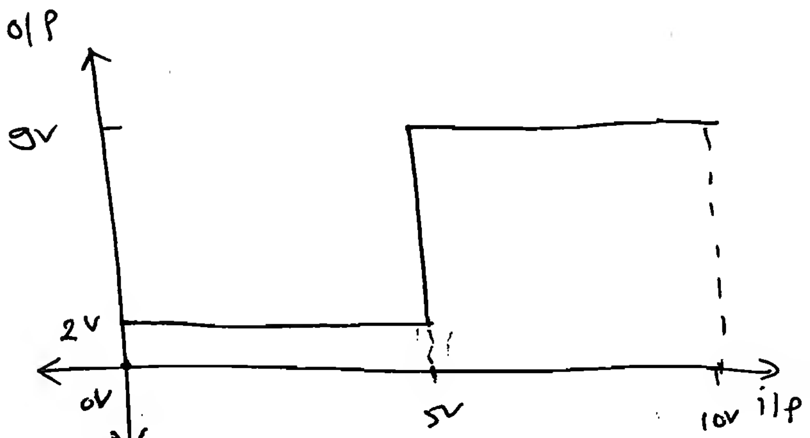


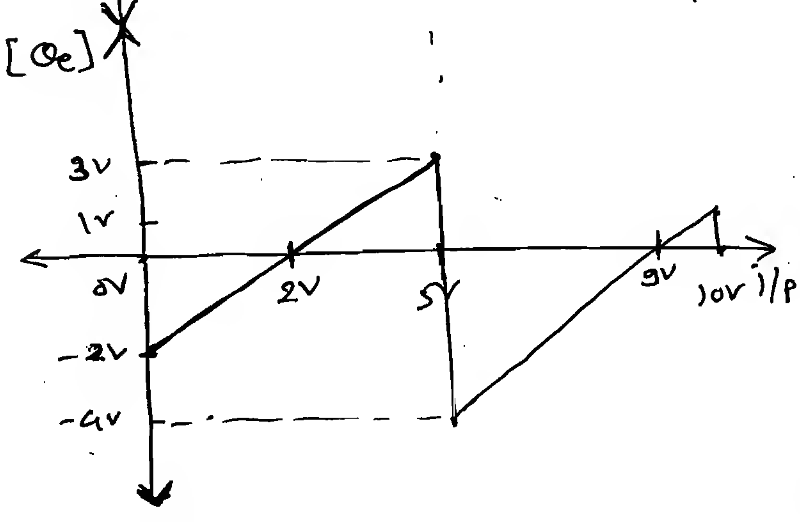
2 $Q_e = S - Q$ = i/p - o/p

S	Q
0	2.5
1	2.5
2	2.5

2







$[Q_e]_{max} = -4V$.

$[Q_e]_{min} = 3V$.

☆ Electrical Representation of Encoder output:

⇒ The encoder is a A to D converter which converts the quantized samples into binary data. In order to represent the binary data electrically the following methods are used.

① ON-OFF signaling
- 1 → +V.
- 0 → 0V.

② NRZ signaling (or) Bipolar signaling
- 1 → +V.
- 0 → −V.

③ RZ signaling
- 1
  - $\frac{T_b}{2}$ → +V.
  - $\frac{T_b}{2}$ → 0V.
- 0 → 0V.

④ Differential Encoding
- 1 → Prev. level
- 0 → $\overline{\text{Prev. level}}$

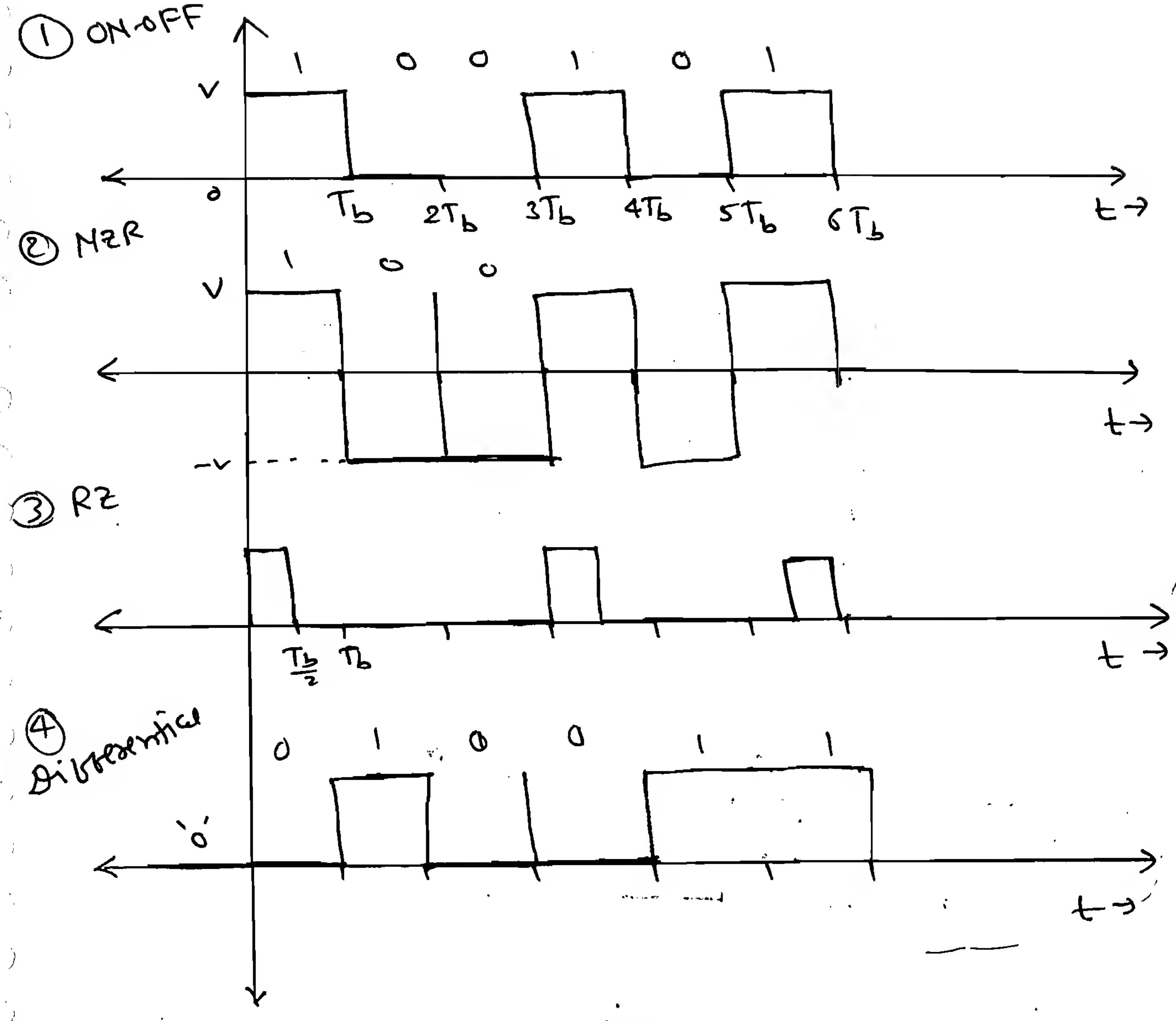
⇒ e.g. 100101

① $\underline{0}$ 010011 → Differential encoded data

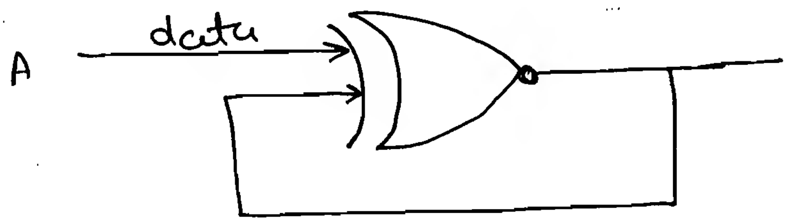
Ref. bit

② 100101

$\underline{1}$ 101100 → Differential encoded data.



⇒ <u>Differential encoding:</u>



⇒ Most widely used method is NRZ which is used in <u>PCM, DPCM, DM, PSK & FSK.</u>

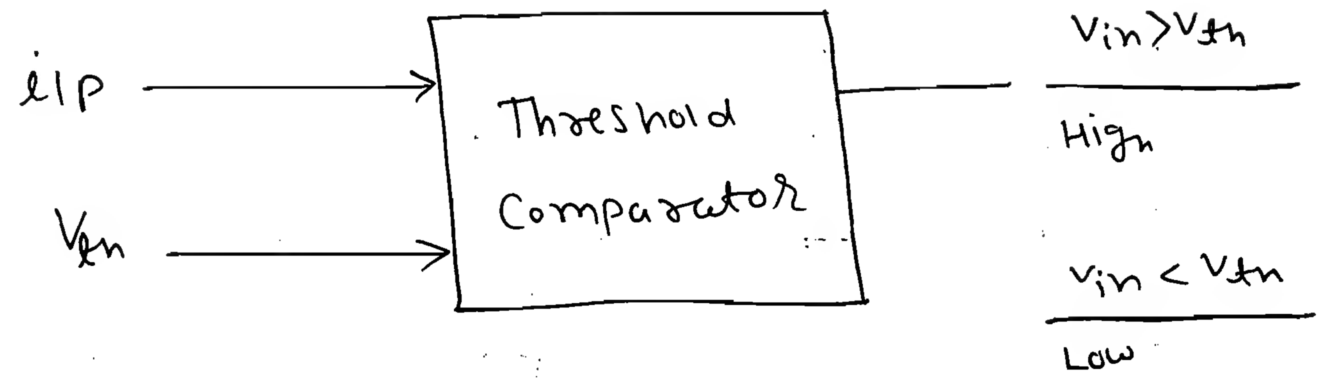
⇒ ON-off signaling is used in ASK.

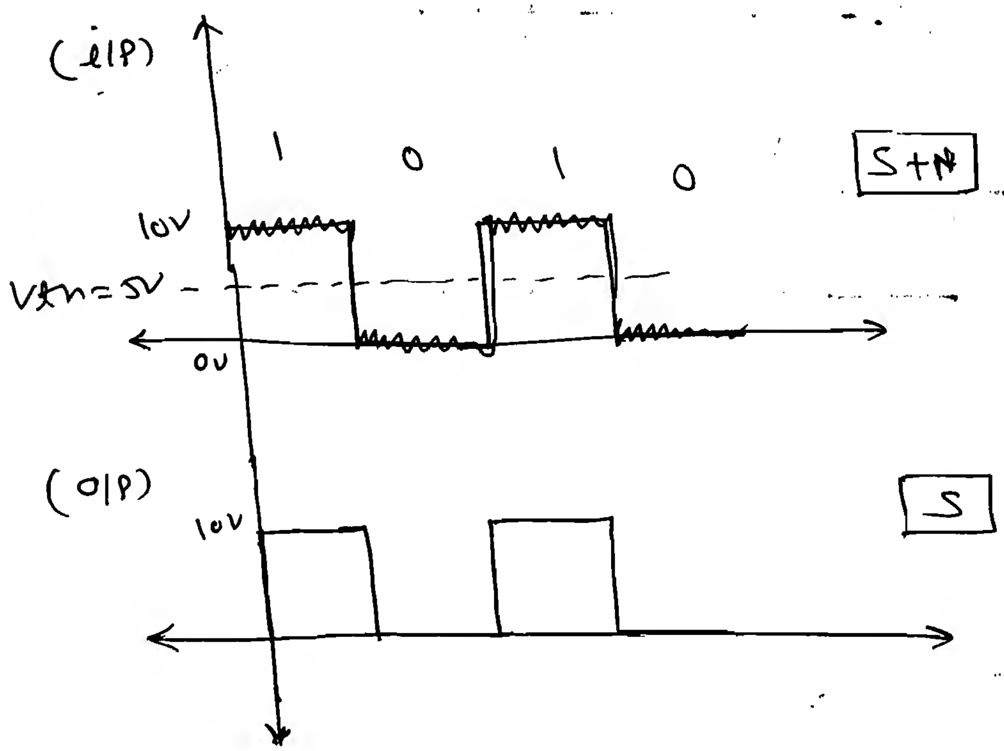
⇒ Differential encoding method is used in DPSK.

⇒ RZ method is used in FOC.

* Regenerative Repeater:

⇒





, Noise is completely removed.

⇒ When effect of noise is very very large it is possible to get an error.

⇒ The input to the regenerative repeater is binary data affected by the noise. If the noise level is very low signal is regenerated. If the Noise level is very high error is occured. But it is ~~negligible in digital communication.~~

Possible to correct the error using Parity bits. So, the effect of noise is negligible in digital communication.

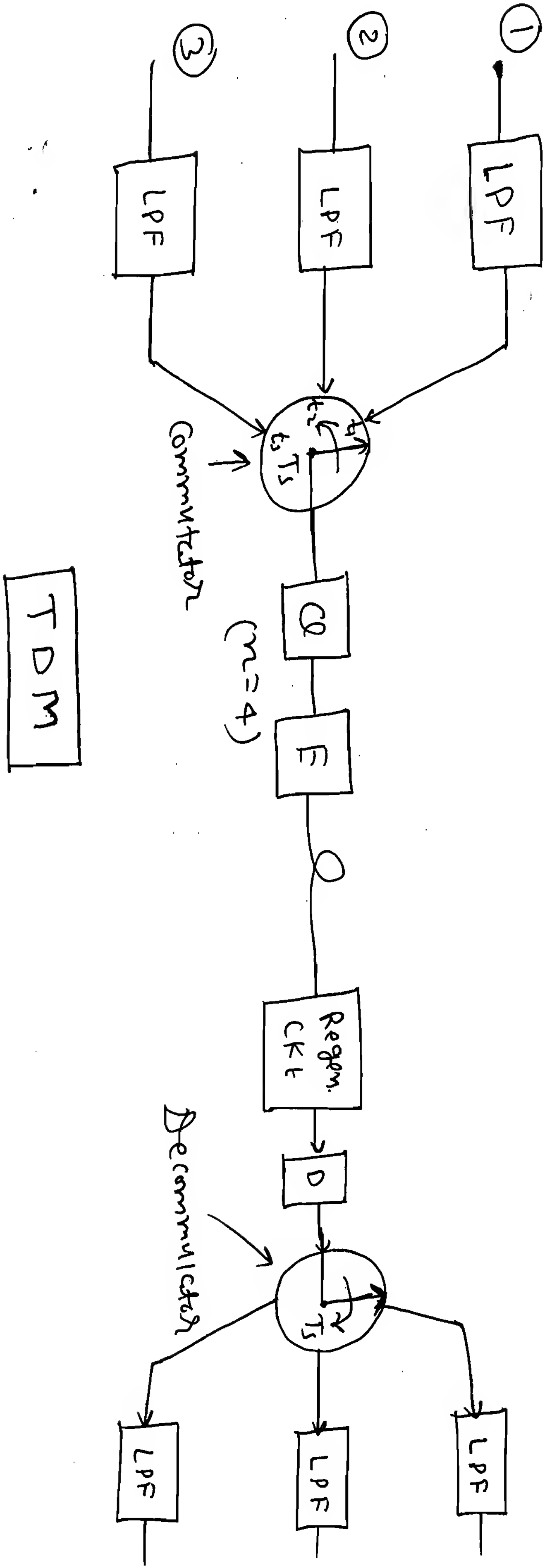
=> At the receiver threshold comparator is used as the regenerative circuit to eliminate the noise at the i/p of receiver.

=> Decoder is a digital to analog converters which converts binary data into samples.

=> The LPF is used to reconstruct the signal from sample. But the i/p to the LPF is the sampled signal with quantization error. Due to this error signal distortion occurs and this distortion is called as the Quantization Noise.

=> To reduced the quantization error the stepsize should be decrease (or) the no. of levels are increase. This is possible only by increasing n. and As n increases the bit rate ($R_b = n \times$ samp. rate), $BW = R_b$ and Probability of error (Pe) also increases.

☆ Time Division Multiplexing:

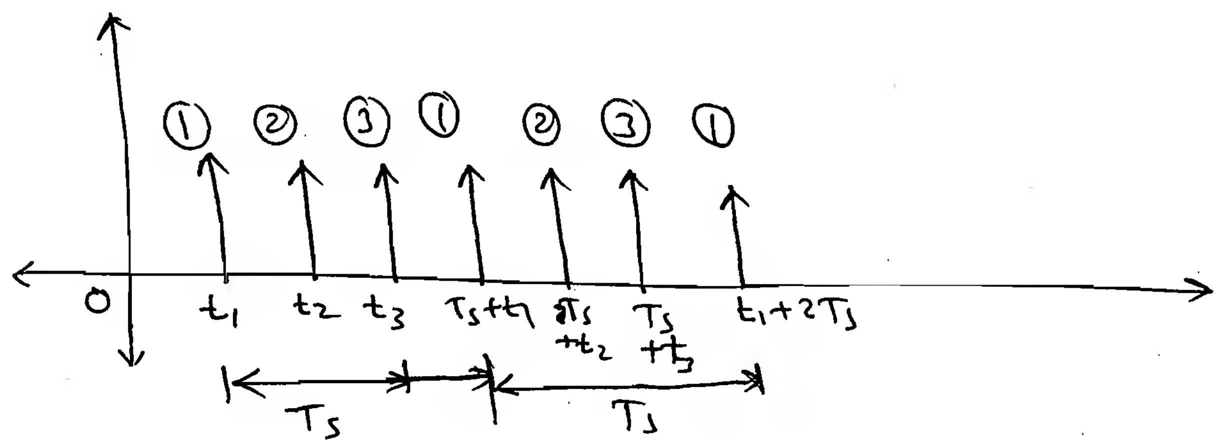


⇒ The LPF is used to eliminate the insignificant high freq.s.

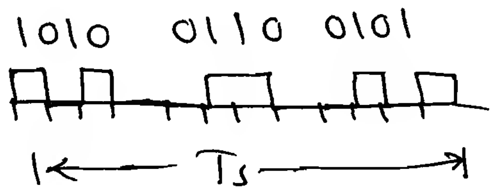
⇒ The Commutator is used to samples the signals for every T_s seconds. The time taken ~~by~~ to complete one revolution of the commutator is T_s seconds which also indicates the sampling intervals.

⇒ In first revolution of the Commutator the first signal is sampled at t_1, second signal is sampled at t_2 and third signal is sampled at t_3.

⇒ The input to the quantizer is shown in figure.



⇒ For every T_s seconds 12 bits are transmitted through the channel.



$$T_b = \frac{T_s}{12} = \frac{T_s}{n \cdot m}.$$

M = no. of Sample (or) no. of signal.

$$\therefore \quad T_b = \frac{T_s}{n \times M}.$$

$$\therefore \quad R_b = \left(\frac{1}{T_b}\right) = \frac{n \times M}{T_s}.$$

$\therefore$ Bit rate of the Multiplexed signal,

$$\boxed{R_b = \frac{1}{T_s} \times n \times M}$$

FDM	TDM
⇒ Carrier freqs are alloted to each signal.	⇒ The time slots are allotted to each signals.
⇒ All the signals are transmitted through the channel at the same time.	⇒ The signals are transmitted in the allotted time only.
⇒ Synchronization is not required.	⇒ Synchronization is required.

$\boxed{Q}$ Five signals each Band limitted to 3 KHz are transmitted through a channel using TDM. Each sample is encoded into 10 bits using PCM. determine the bit rate (or) the multiplexed signals.

Soln: $M = 5$, $f_m = 3KHz$, $n = 10$

$\therefore$ Ny. rate $= 2f_m = 6000$ sample/sec.

$\therefore$ sampling rate $= \frac{1}{T_s} = 6000$ sample/sec.

Now, $R_b = \frac{1}{T_s} \times n \times M$

$R_b = 6000 \times 10 \times 5.$

$\boxed{R_b = 300 \text{ KbPS}}$

$\boxed{Q}$ 10 Voice signals are transmitted through a channel using TDM. Each sample is encoded into 8 bits. The time taken to complete one revolution of the commutator is 125 µs. Determine the bit rate of the multiplexed signal.

Ans: $M = 10$, $n = 8$

$T_s = 125\ \mu s.$

$\therefore$ Bit rate $R_b = \frac{1}{T_s} \times n \times M.$

$\therefore R_b = \frac{10^6}{125} \times 8 \times 10$

$$\therefore R_b = \frac{10^6}{125} \times 10 \times 8$$

$$= 8000 \times 10 \times 8$$

$$\boxed{R_b = 640 \text{ kbps}}$$

$\Rightarrow$ To Synchronize the Commutator and decommutator the binary data will be transmitted in the form of frames. The no. of bits generated in one revolution is Considered as one frame. If the duration of the frame is T_s sec. and a frame consist of data bits and synchronization bits.

$\Rightarrow$

Frame

$\boxed{nM + a}$

$|\leftarrow T_s \rightarrow|$

$$T_b = \frac{T_s}{12 + 5}$$

$$T_b = \frac{T_s}{nm + a}.$$

$$\Rightarrow \quad \therefore R_b = \frac{(nm + a)}{T_s}.$$

$$\Rightarrow \quad \therefore \boxed{R_b = \frac{1}{T_s} \times (nm + a)} \text{ bps.}$$

Q 8 signals are transmitted through a channel using TDM. Each samples is encoded into 10 bits. The speed of the Commutator is 5000 revolution/sec. Determine the bit rate of the multiplexed signal.

① If Synchronization requires 5 extra bits per frame.

② If Synchronization requires 1 extra bits per sample.

Soln: $m = 8$, $n = 10$, $\frac{1}{T_s} = 5000$ rev/sec.

① $a = 5$

$$R_b = \frac{1}{T_s} \times (mn + a)$$

$$= 5000\ (80 + 5).$$

$$\boxed{R_b = 425 \text{ KBPS.}}$$

② $a = 1$

$$R_b = \frac{1}{T_s} \times (mn \times a).$$

$$= 5000 \times (81).$$

$$\therefore \boxed{R_b = 405 \text{ KBPS.}}$$

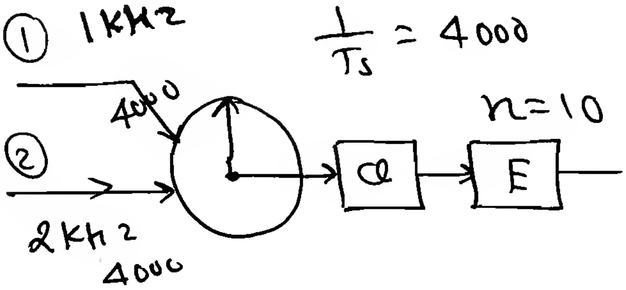
*

→ If the freqs are different the following two methods are used.

Method - 1

$\frac{1}{T_s} = 4000$

$n = 10$



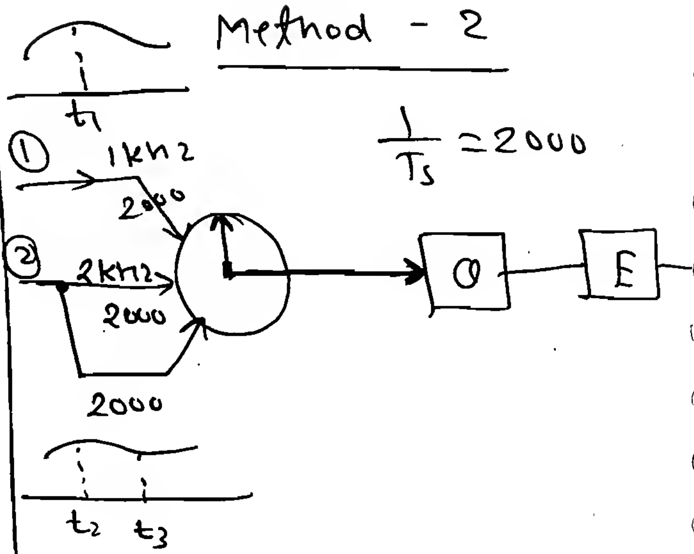
$R_b = \frac{1}{T_s} \times n \times M$

$= 4000 \times 10 \times 2$

$\boxed{R_b = 80 \text{ kbps}}$

Method - 2

$\frac{1}{T_s} = 2000$



$R_b = \frac{1}{T_s} \times n \times M$

$= 2000 \times 10 \times 3$

$\boxed{R_b = 60 \text{ kbps}}$

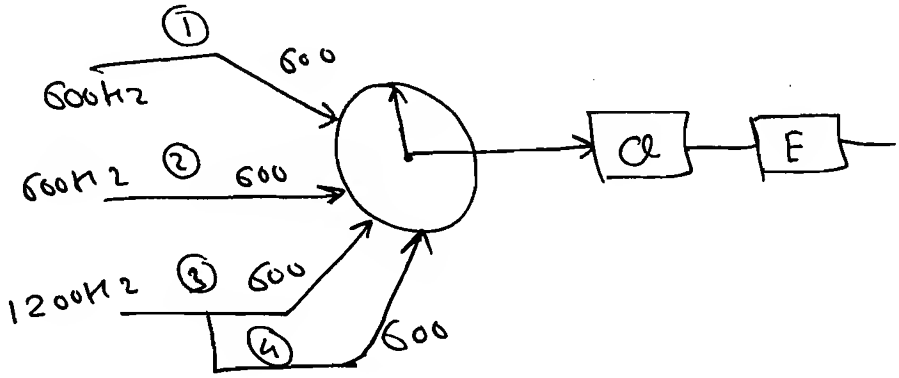
⇒ The bit rate is reduced in second method. As $\boxed{R_b \downarrow \Rightarrow BW \downarrow \Rightarrow P_e \downarrow}$

⇒ The second method is applicable only the second signal has the multiple integer of the least frez.

$\boxed{Q}$ Three signals bandlimited to 600 Hz, 600 Hz and 1200 Hz. are sampled at nyquist rate and transmitted through a channel using TDM. Each sample is encoded into 12 bits. Determine the R_b of

Multiplexed signals.

Soln:



$\therefore R_b = \frac{1}{T_s} \times n \times M = 1200 \times 12 \times 4$

$= 57.6$ Kbps.

Q Repeate the above numerical Problem if the signals are Band limitted to 600 Hz, 1200 Hz & 1800Hz.

Soln:

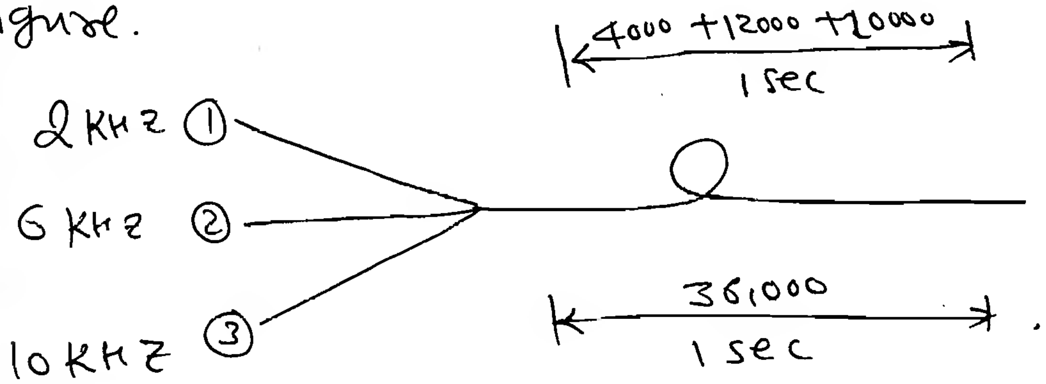
$R_b = \frac{1}{T_s} \times n \times M$

$= 1200 \times 6 \times 12$

$\therefore \boxed{R_b = 86.4 \text{ KbPS.}}$

* ⇒ Consider a signal which is Band limited to 5 KHZ and assume the samples are transmitting through a channel. In order to transmitt 10,000 samples in 1sec the minimum BW of channel required is 5 KHZ.

⇒ Now, Assume that three signals are multiplexed and samples transmitted through the channel as shown in the figure.

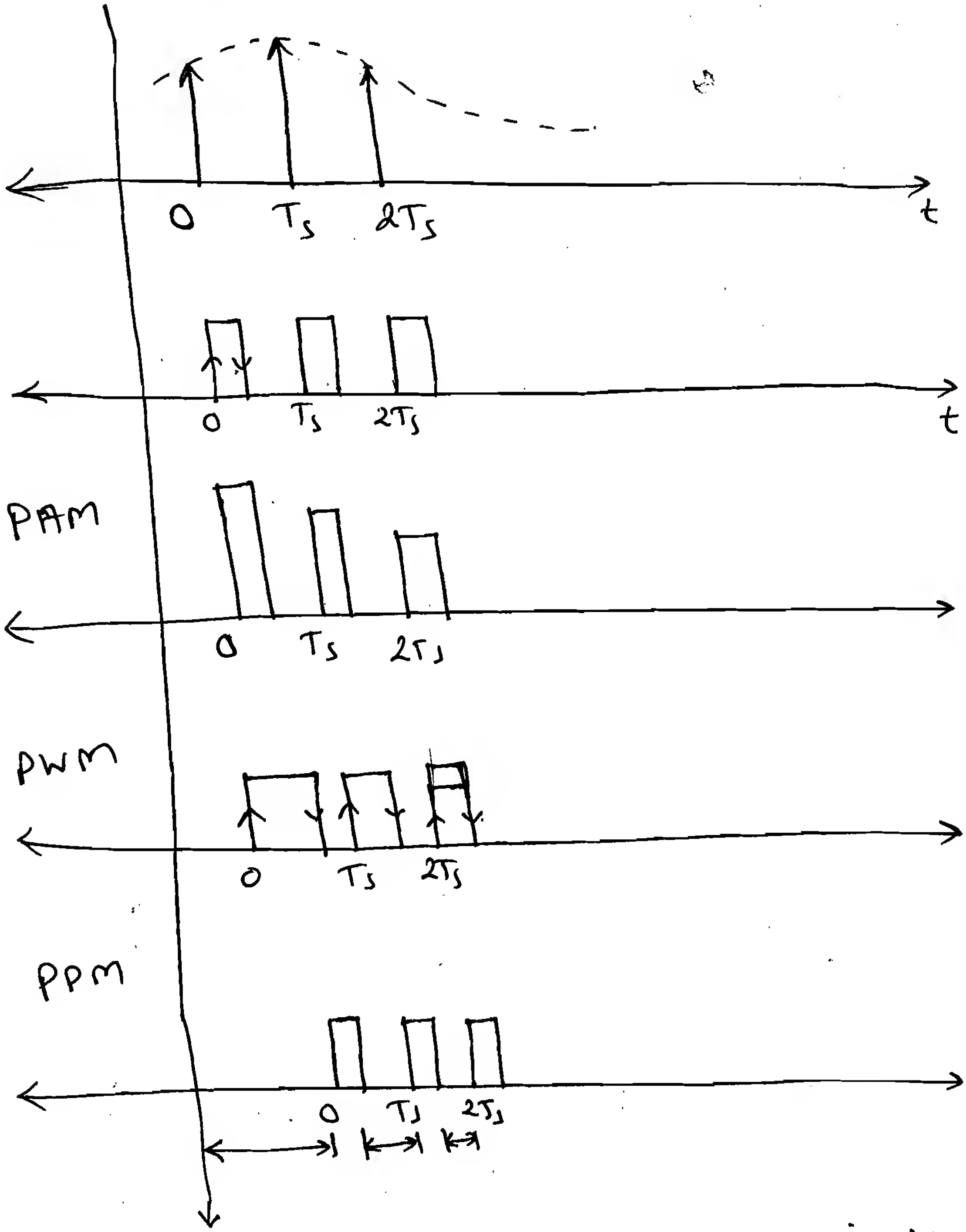


⇒ In the above example, sampling rate of the multiplexed signals is 36,000 per sec.

⇒ To transmitt 36,000 samples in one seconds the minimum BW of the channel required is 18 KHZ.

⇒ In Pulse Analog Communication a Parameter of the Rectangular pulses is Varied according to the sampled value.

⇒ In PAM the Power is variable and the BW is Constant.



⇒ In PWM Power is variable and BW is Varu.

⇒ In PPM the Power is constant and the BW is also Constant.

* T1 Carrier System :-

⇒ T_1 Carrier System is used in telephone exchange to multiplex voice signal using TDM.

⇒ In telephone transmission 24 voice signals are multiplexed to form a T_1 system.

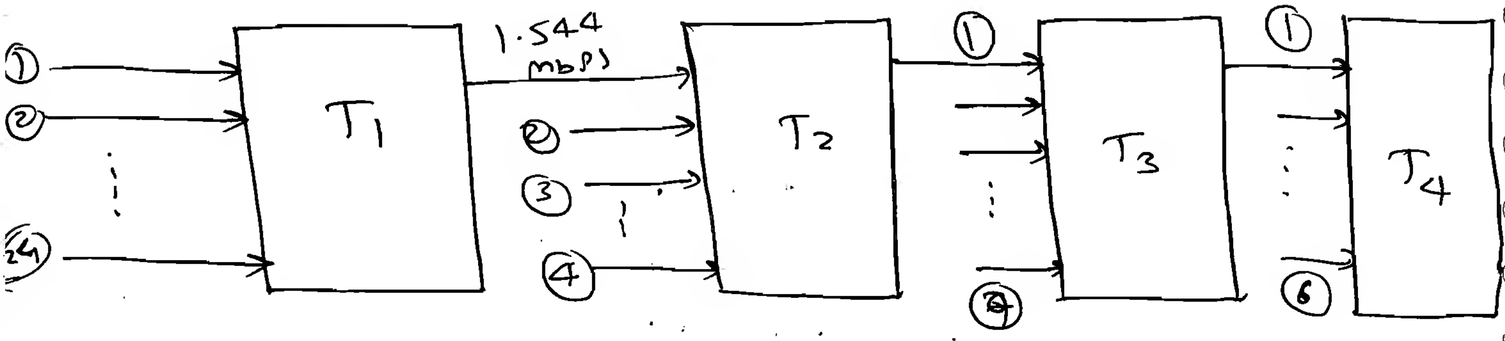
⇒ $\frac{1}{T_s} = 8000$, $n = 8$

$$R_b = \frac{1}{T_s} \times [mn + a].$$

$$R_b = 8000 \times [8 \times 24 + 1].$$

$$\boxed{R_b = 1.544 \text{ mbps.}}$$

* Digital multiplexer:



24x4 = 96

24 x4 x7

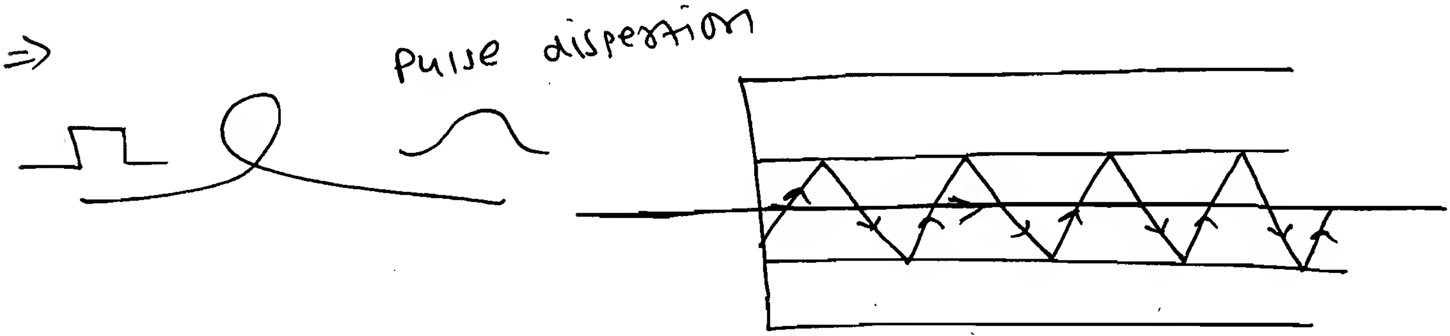
24x4x7 x6.

☆ Inter Symbol Interference :- (ISI):

⇒ When a Rectangular pulse is transmitted through a fiber optic cable, pulse dispersion occurs. In a fiber optic cable light signal is propogated through a core material by a mechanism called Total internal reflection.

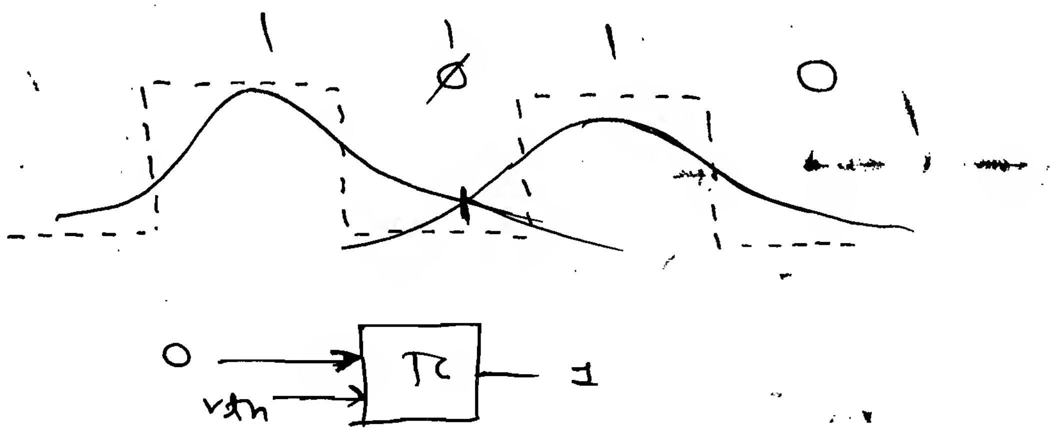
⇒ Due to variations in the propogation times of the light rays pulse dispersion occurres.

⇒



⇒ Assume that the binary data 1010 is transmitted through a fiber optic cable. Due to pulse dispersion the signal at the o/p will as shown in fig.

⇒



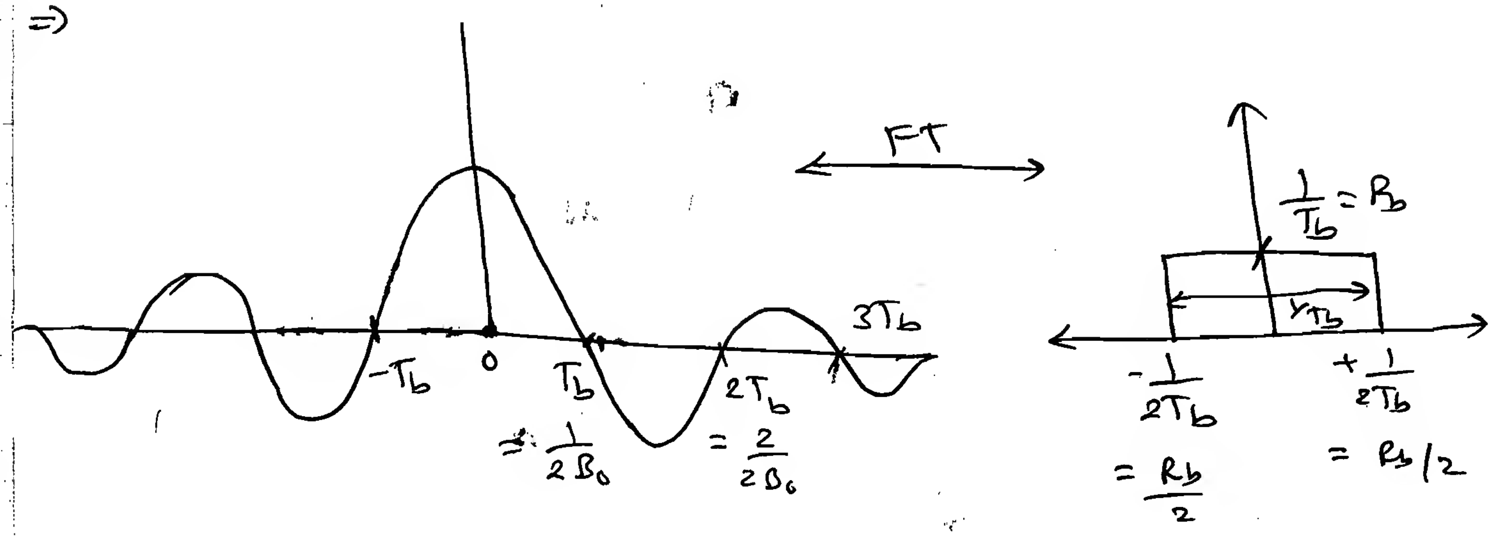
⇒ Due to ~~this~~ pulse dispersion, over lapping occurs and this phenomena is called as the <u>Inter Symbol Interference</u>. Due to this overlapping (or) ISI errors will occured.

⇒ To overcome the ISI pulse shapping is required.

⇒ To overcome the problem raised cosine pulse is used instead of rectangular pulses.

⇒ To overcome the ISI, sinc f^n is used in time domain. But the sinc f^n should be design in such a way that the signal amplitude should be very high at $t=0$. and $T_b, 2T_b, 3T_b, \ldots$ the value should be zero.

⇒



$p(t) = \text{sinc}\,[R_b \cdot t]$

$p(t) = AT\,\mathrm{sinc}(tT)$

here $T = R_b$, $AT = \frac{1}{R_b} \times R_b = 1$.

So, $\boxed{p(t) = \mathrm{sinc}(R_b \cdot t)}$.

$\Rightarrow$ BW of the sinc f^n is $R_b/2$.

So, the minimum BW of the channel required is $R_b/2$.

$$\boxed{B_0 = R_b/2 \text{ Hz}} \leftarrow \text{N.B}$$

$$\Rightarrow \boxed{\begin{array}{l} R_b = 2B_0. \\ \hline T_b = \frac{1}{2B_0}. \end{array}} \leftarrow \text{N.B.}$$

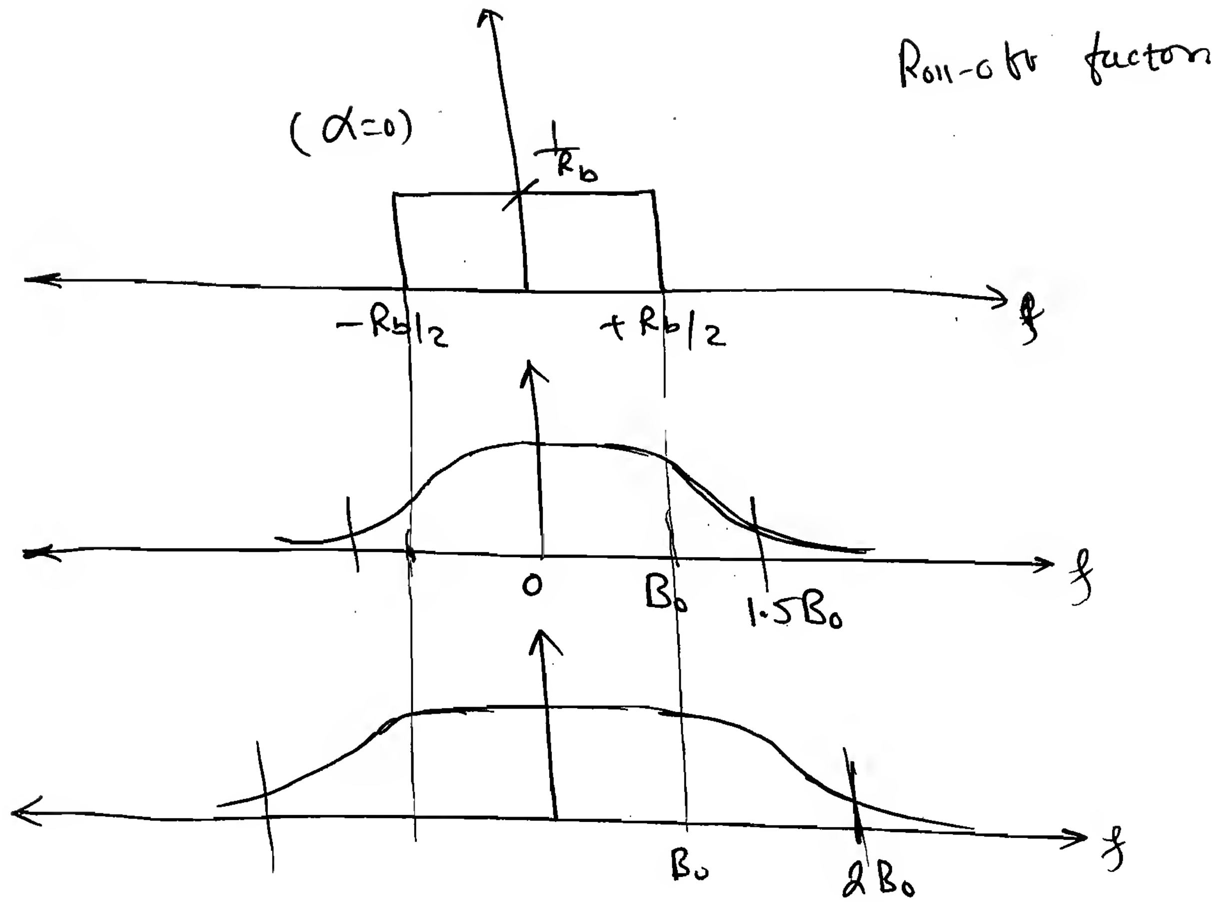
$\Rightarrow$ $p(t) = \mathrm{sinc}[2B_0 t]$

$$\boxed{p(t) = \frac{\sin[2\pi B_0 t]}{2\pi B_0 t}} \leftarrow \text{N.B}$$

$\Rightarrow$ It is not possible to generate a Rectangular pulse in freq. domain.

$\Rightarrow$ In the Practical case it is not sudden transition from one level to another level in freq. domain.

⇒



⇒ The transmission BW of the Raised Cosine f^n is.

$$\boxed{B_T = B_0[1+\alpha].}$$

α = Roll-off factor.

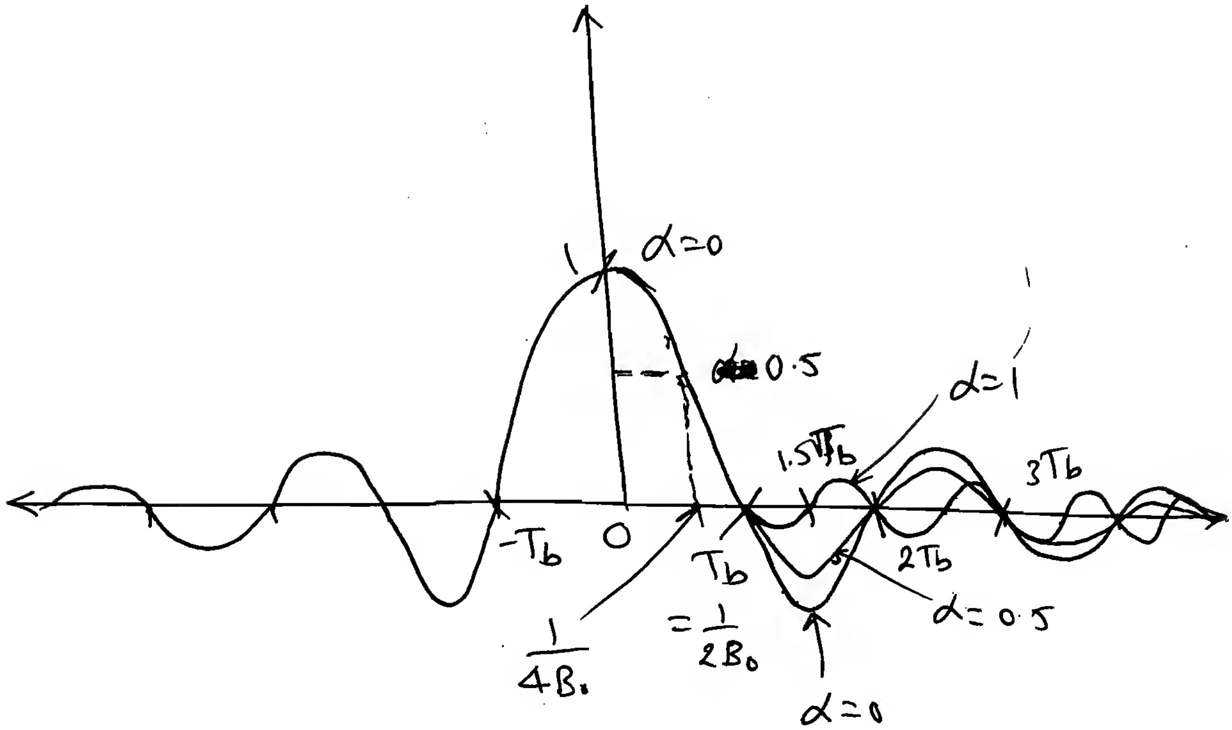
⇒ When $\alpha = 0$, $B_T = B_0 = \frac{R_b}{2} = \frac{1}{2T_b}$. So, the minimum BW required to transmitt the signal is $R_b/2$.

⇒ When $\alpha = 1$.

$$B_T = B_0[1+1]. = 2B_0 = \frac{2.R_b}{2}$$

$\boxed{B_T = R_b}$ (Practical case)

⇒



⇒ $\alpha = 0 \rightarrow p(t) = \text{sinc}[2B_0 t]$

⇒ $0 < \alpha < 1 \rightarrow p(t) = \dfrac{\text{sinc}[2B_0 t] \cdot \cos 2\pi B_0 t \alpha}{1 - 16\alpha^2 B_0^2 t^2}$

⇒ $\alpha = 1 \rightarrow p(t) = \dfrac{\text{sinc}[2B_0 t]}{1 - 16 B_0^2 t^2}$

Q

→ The data o/p in a digital Communication system 50 KBPS. To transmit the binary data without ISI. Determine the BW of the channel required.

(i) $\alpha = 0$.

(ii) $\alpha = 0.5$

(iii) $\alpha = 1$.

Soln: Here, $R_b = 50$ KBPS.

$$\therefore B_0 = \frac{R_b}{2} = 25 \text{ KHZ}.$$

(1) $\alpha = 0.$

$$B_T = B_0[1+\alpha] = B_0[1+0] = B_0$$

$$\boxed{B_T = 25 \text{KHZ}}$$

(2) $\alpha = 0.5$

$$B_T = B_0[1+\alpha] = B_0[1+0.5]$$

$$B_T = 1.5 B_0$$
$$= 1.5 \times 25 K$$

$$\boxed{B_T = 37.5 \text{ KHZ}}$$

(3) $\alpha = 1$

$$B_T = B_0[1+\alpha] = B_0[1+1]$$

$$B_T = 2 B_0$$

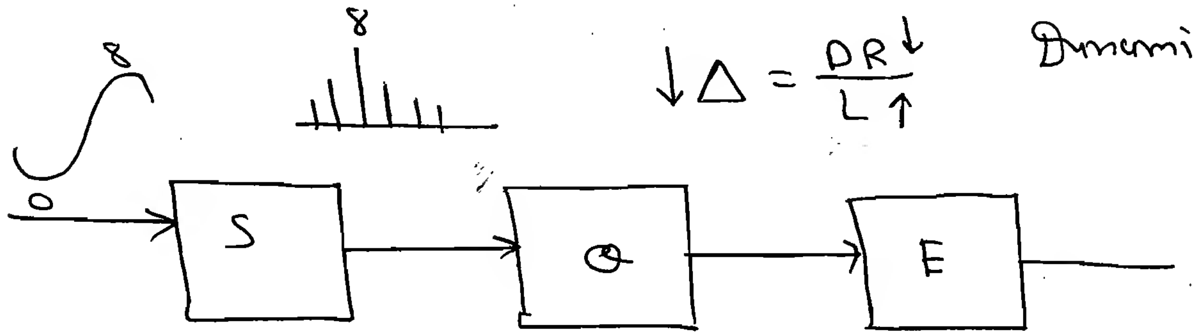
$$\therefore \boxed{B_T = 50 \text{ KHZ}}$$

☆ DPCM (Differential Pulse Code Modulation):

⇒ DPCM is used to reduced the quantization error without increasing the no. of bits.

⇒ Consider a PCM system and assume that the i/p signal varies from 0 to +8V.

$\downarrow \Delta = \frac{DR \downarrow}{L \uparrow}$ Dynamic range



$n = 2$ $\quad \Delta = \frac{8-0}{4} = 2V$ $\quad [Q_e]_{max} = 1V$

$n = 3$ $\quad \Delta = \frac{8-0}{8} = 1V$ $\quad [Q_e]_{max} = 0.5V$

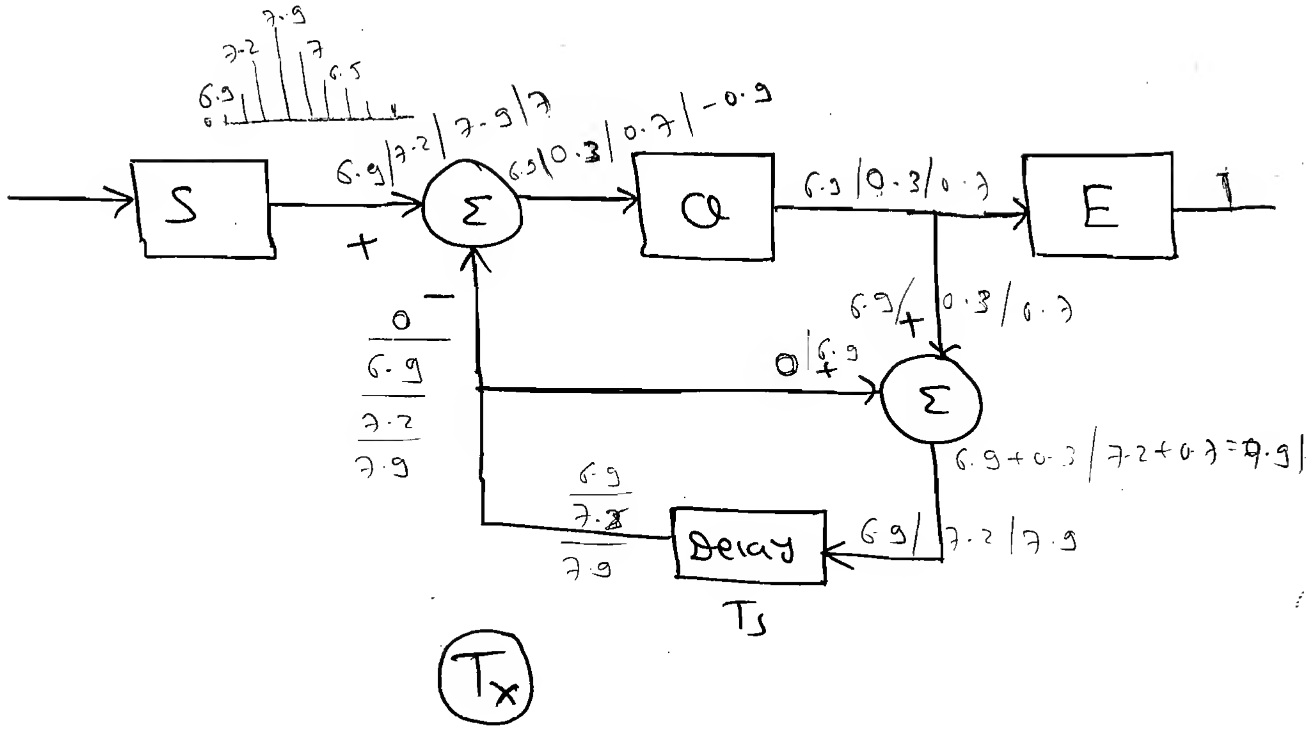
$n = 4$ $\quad \Delta = \frac{8}{16} = 0.5V$ $\quad [Q_e]_{max} = 0.25V.$

⇒ Quantization error depends on the step size. In a PCM system, the no. of levels are increased to reduced the step size.

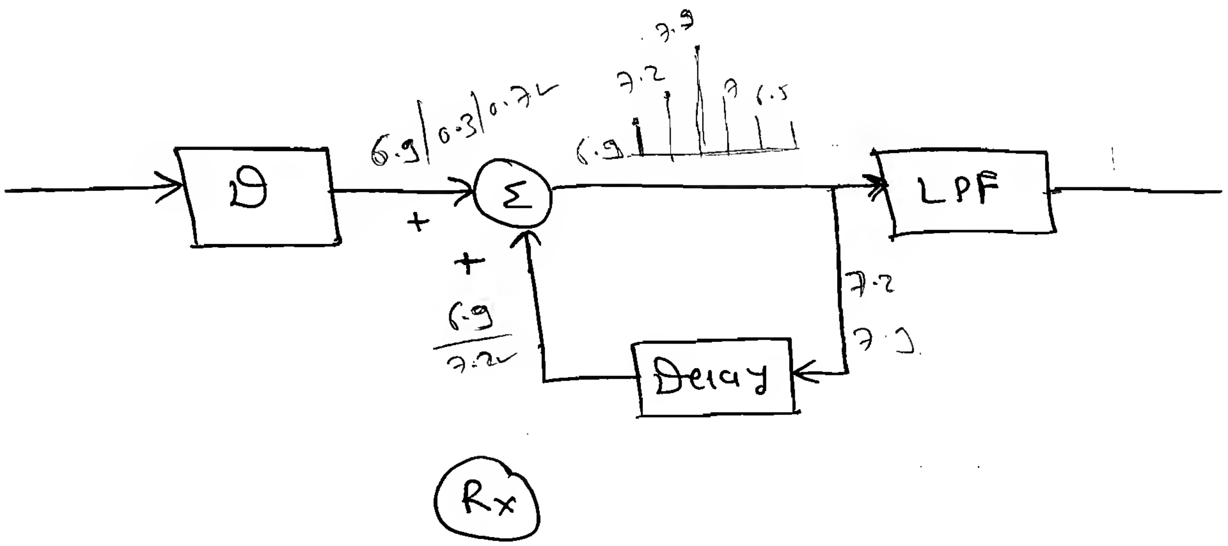
⇒ In DPCM, dynamic range of the quantizer is reduced to reduce the step size.

* Block Diagram of DPCM Tx:

=>



* Block Diagram of DPCM Rx:



=> In a PCM system the samples are applied directly to Quantizer. So, the dynamic range is very high.

=> In a DPCM system the difference betn two successive samples is applied as the input to the Quantizer to reduced the

dynamic range.

⇒ In a PCM system, the dynamic Range is independent Sampling rate.

⇒ In a dpcm system, the Dynamic Range varies with the Sampling rate.

⇒ The hardware Complexity of a Dpcm system is very high when Compared with PCM.

⇒ The bit rate of the Dpcm signal is Same as the pcm.

⇒ In a pcm system, each sample is encoded into n-bits after quantization. This n bits are transmitted through the channel in T_s seconds.

⇒ In Dpcm, the difference betn two ~~steps~~ successive Sample is quantized and encoded into n-bits. So, the Bit rate of the Dpcm signal is same as the pcm.

$P_t \downarrow$ AM, DSB

$BW \downarrow$ SSB

$Q_e \downarrow$ PCM, DPCM

$BW, R_b \downarrow$ DM

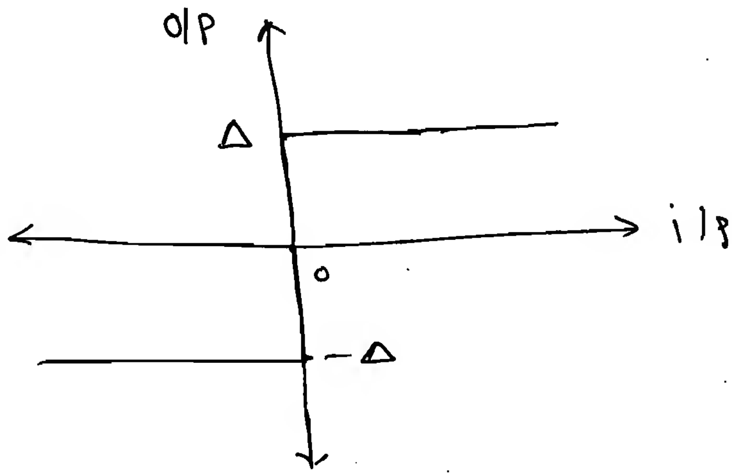
☆ Delta Modulation : (1 Bit DPCM).

⇒ Delta Modulation is used to reduce R_b (or) BW of the signal.

⇒ Delta modulator is consider as 1 bit DPCM system. So, the encoder is a 1 bit A to D converter.

⇒ Two quantization levels are used which are $+\Delta$ and $-\Delta$. When the i/p to the quantizer is +ve the o/p is Δ otherwise the o/p is $-\Delta$.

⇒ The transfer characterestics of the quantizer is shown in figure:



⇒ In Delta modulation, the no. of samples and the no. of bits are same as $n=1$,

Bit rate = Sampling rate × n

But $n=1$

So, Bit rate = Sampling rate.

$$\boxed{R_b = \text{Sampling rate}} \leftarrow \text{H.B.}$$

* PCM, DPCM

$$R_b = \left(\frac{1}{T_s} \times n\right) \text{ bps.}$$

* DM (n=1)

$$R_b = \frac{1}{T_s} = \text{Pulse rate.}$$

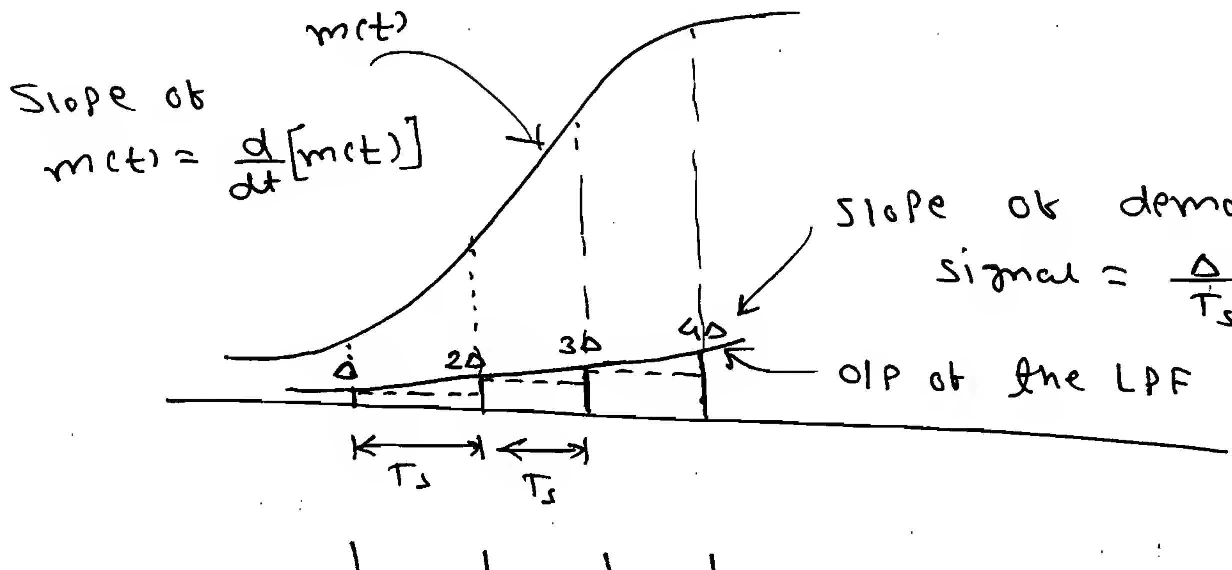
⇒ In PCM and DPCM n-bits are transmitted through the channel for every T_s seconds. So, $\boxed{T_b = T_s/n}$

⇒ In DM, only one bit is transmitted through the channel for every T_s seconds. So, the pulse width is more in DM when compared with PCM & DPCM.

⇒ At the receiver binary symbol '1' is decoded as 'Δ' and '0' is decoded as −Δ. So, the input to the LPF is increases and decreases in steps of Δ. The signal construction depends on the value of Δ.

* Optimum Value of Δ:

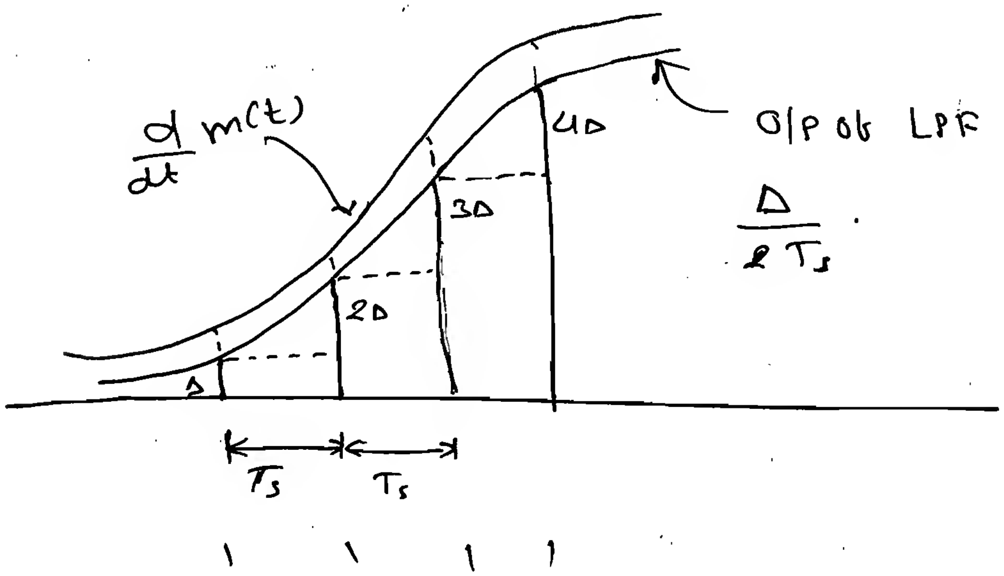
⇒ ① Assume that the Δ is very small.



if $\boxed{\frac{\Delta}{T_s} < \frac{d}{dt} m(t)}$ → Slope over load.

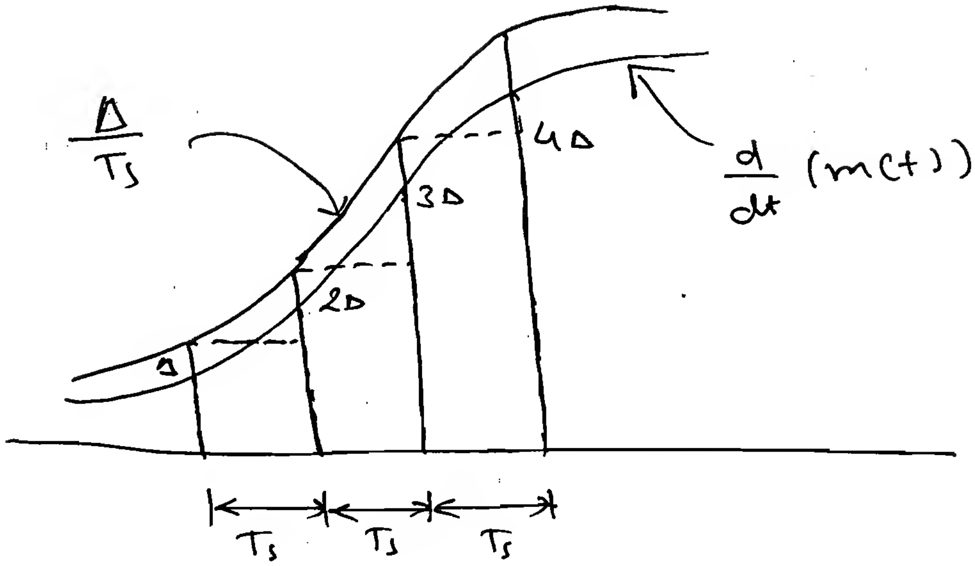
H.B

⇒ ② Assume that the Δ is large.



$$\therefore \frac{\Delta}{T_s} = \frac{d}{dt}(m(t)).$$

③ Assume that Δ is very large,



$\therefore$ $\boxed{\frac{\Delta}{T_s} > \frac{d}{dt} m(t).}$ $\longrightarrow$ Granular Noise. ← H.B

⇒ So, the optimum value of Δ is given by,

$$\Rightarrow \boxed{\Delta = \frac{\frac{d}{dt} m(t)}{1/T_s}} \leftarrow \text{H.B}$$

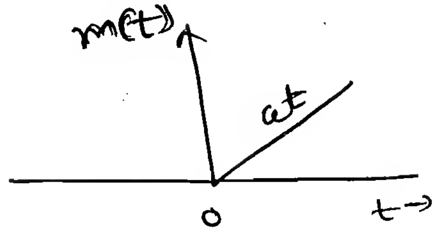
$$\boxed{\Delta = \frac{\text{Slope of the message signal}}{\text{Sampling rate.}}}$$

⇒ Granular Noise Power $= \frac{\delta^2 B}{3 f_s}$

$$\boxed{= \frac{\Delta^2 B}{3 f_s}} \leftarrow \text{H.B.}$$

Case-(i):

→ Assume that the message is a ramp signal $m(t) = at$



Slope $= \frac{d}{dt} m(t) = a$

$\therefore \boxed{\Delta = \frac{a}{\text{samp. rate}}}$

$\boxed{a}$ Input to the DM is $m(t) = 5t$ and the sampling rate is 5000 samp/sec. Determine the step size.

Ans: here, $m(t) = 5t$, $a = 5$.

Sampling rate = 5000 samp/sec.

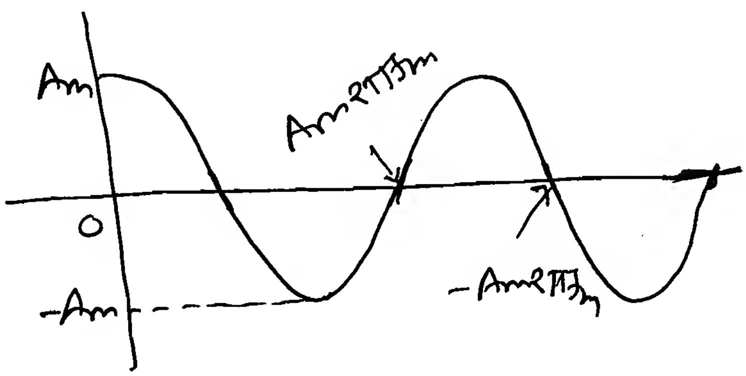
$\therefore \quad \Delta = \frac{a}{\text{samp. rate}}$

$\Delta = \frac{5}{5000} = \frac{1}{(1000)}$

$\therefore \boxed{\Delta = 1\,mV}$

case-(ii)

→ when $m(t) = A_m \cos 2\pi f_m t$



Slope of $m(t)$

$= \frac{d}{dt} m(t)$

$= -A_m 2\pi f_m \sin 2\pi f_m t$

$\therefore \boxed{\Delta = \frac{A_m 2\pi f_m}{1/T_s}}$ ← H.B.

$\boxed{Q}$ The input to the DM $5\cos 1000t \times 2\pi$. The pulse rate is 36,000 pulses/sec. Determine the step size.

Ans:

$$m(t) = 5\cos 1000t \times 2\pi$$

$$A_m = 5, \quad 2\pi f_m = 2\pi \times 1000$$

$$\therefore \quad \Delta = \frac{A_m 2\pi f_m}{1/T_s}.$$

Pulse rate = bit rate = Sampling rate

$= 36,000.$

$$\therefore \quad \Delta = \frac{5 \times 2 \times \pi \times \cancel{1000}}{\cancel{36000}\ 36}$$

$$\boxed{\Delta = \frac{10\pi}{36}.}$$

$\boxed{Q}$ The input to the DM is a sinusoidal signal whose freq. varies from 200 Hz to 4000 Hz. The Sampling rate is 8 times the nyquist rate. The peak amplitude of the signal is 1V. Determine the stepsize when the signal freq. is 800 Hz.

Ans:

$f_m = 4000$ Hz.

$\therefore$ Sampling rate $= 8 \times$ nyquist rate

$= 8 \times 2f_m.$

Samp. rate = $8 \times 2 \times 4000$

$= 64000$ sample/sec.

$$\therefore \Delta = \frac{A_m 2\pi f_m}{1/T_s}$$

$$\therefore \Delta = \frac{1 \times 2\pi \times \cancel{4000}^{\,500}}{\cancel{64000}\ 80}$$

$$\therefore \boxed{\Delta = \frac{\pi}{40}\ V}$$

Q The input to the DM is $m(t) = A_m \cos 2\pi f_m t$. The stepsize $\Delta = 0.628V$ and the sampling rate is 40,000 sample/sec the combination of sinusoidal signal the amplitude and freq. for which the slope overload distortion occurs.

A	A_m	F_m
(A)	0.3 V	8 KHz
(B)	1.5 V	4 KHz.
(C)	3 V	1 KHz.
(d)	1.5 V	2 KHz.

Soln:

$\Delta = 0.628$ V , $\frac{1}{T_s} = 40,000$ samples/sec.

for slope over distortion

$$\frac{\Delta}{T_s} < \frac{d}{dt} m(t).$$

$$\Rightarrow \frac{\Delta}{T_s} < A_m 2\pi f_m.$$

∴ $A_m f_m > \Delta \times \frac{1}{T_s} \times \frac{1}{2\pi}$

$A_m . f_m > \frac{0.\cancel{628}}{\frac{1}{\cancel{10}}} \times \frac{\cancel{40,000}}{4000} \times \frac{1}{2\pi}$

∴ $A_m . f_m > 4000$

So, Ans is (B) ~~because~~ because

$A_m . f_m = 1.5 \times 4 = 6000 > 4000.$ ✓

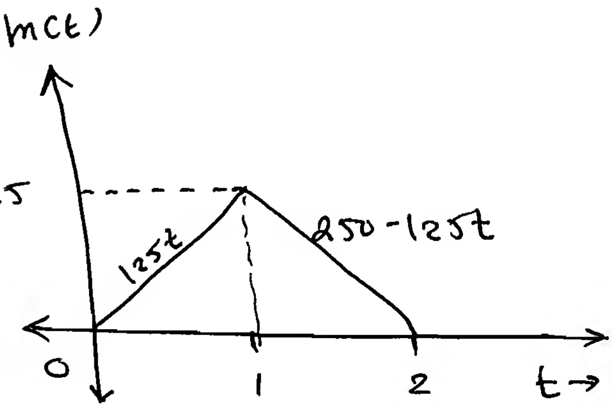
$\boxed{Q}$ Consider a message signal which is apply to a DM. ~~m(t)~~ = 12

$m(t) = 125t\left[u(t) - u(t-1)\right] + (250 - 125t)\left[u(t-1) - u(t-2)\right]$

The Sampling rate is 32,000 samp. per seconds

Determine step size.

Soln:



$\left|\frac{d(m(t))}{dt}\right|_{max} = 125$

$\frac{1}{T_s} = 32,000$

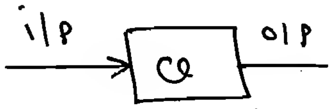
∴ $\Delta = \frac{\left|\frac{d}{dt} m(t)\right|_{max}}{1/T_s}$

∴ $\Delta = \frac{125}{32,000}$

$\boxed{\Delta = 2^{-8} . V}$

⇒ In delta modulation the stepsize Δ is Const~~act~~ant and depends on the maximum slope. But the stepsize Δ should ~~should~~ be varied whenever the slope of signal changes.

⇒ In <u>Addeptive</u> <u>Delta</u> <u>Modulation</u> the stepsize is varied according to the slope of the message signal.

⇒ 

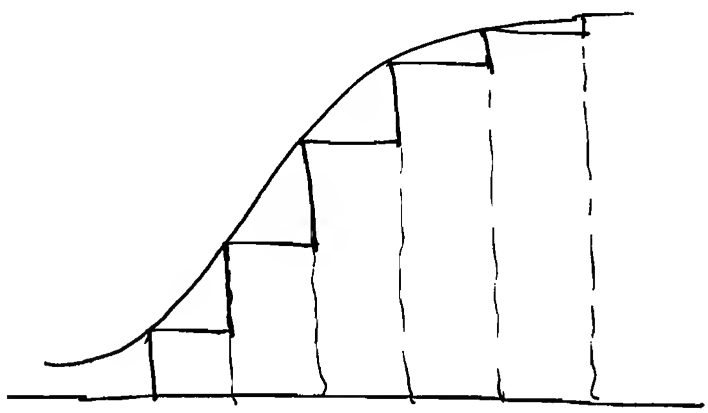
$\Delta \propto$ Slope of the m(t).

$\therefore \Delta \propto$ (present sample − preveious sample).

$\Delta \propto$ i/p

(*)

(DM)



(ADM)

☆ Band Pass Data Transmission (OR)

Digital Carrier Modulation:

⇒ The output of the encoder in a Base Band system is binary data which is having significant low freq. So, it is not possible to transmitt binary data directly into the free space. A high freq. Carrier signal is used to transmitt the data into free space.

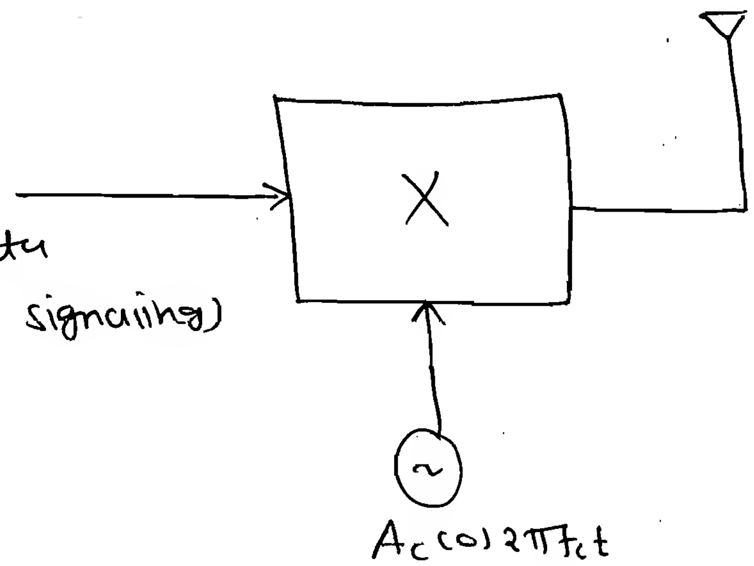
⇒ The three parameters of the carrier which can be varied according to the digital signal are Amplitude, Frequinces & phase. Therefore, the three modulation techniques are,

1. Amplitude Shift Keying (ASK).
2. Frequency shift keying (FSK).
3. Phase shift keying (PSK).

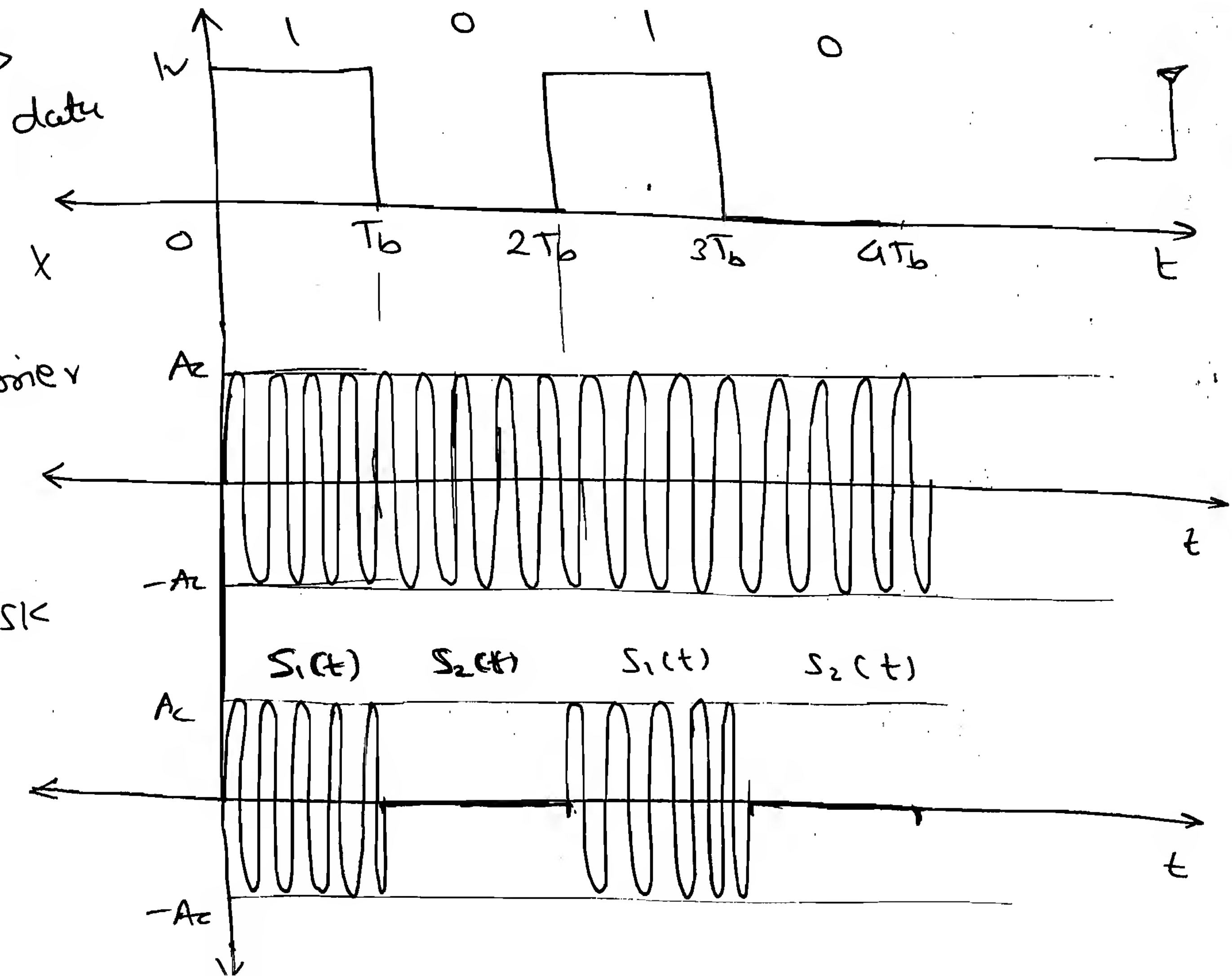
① Amplitude Shift Keying:

⇒ The binary data in ON-OFF signaling is multiplied with the carrier to generate the ASK signal.

⇒



⇒

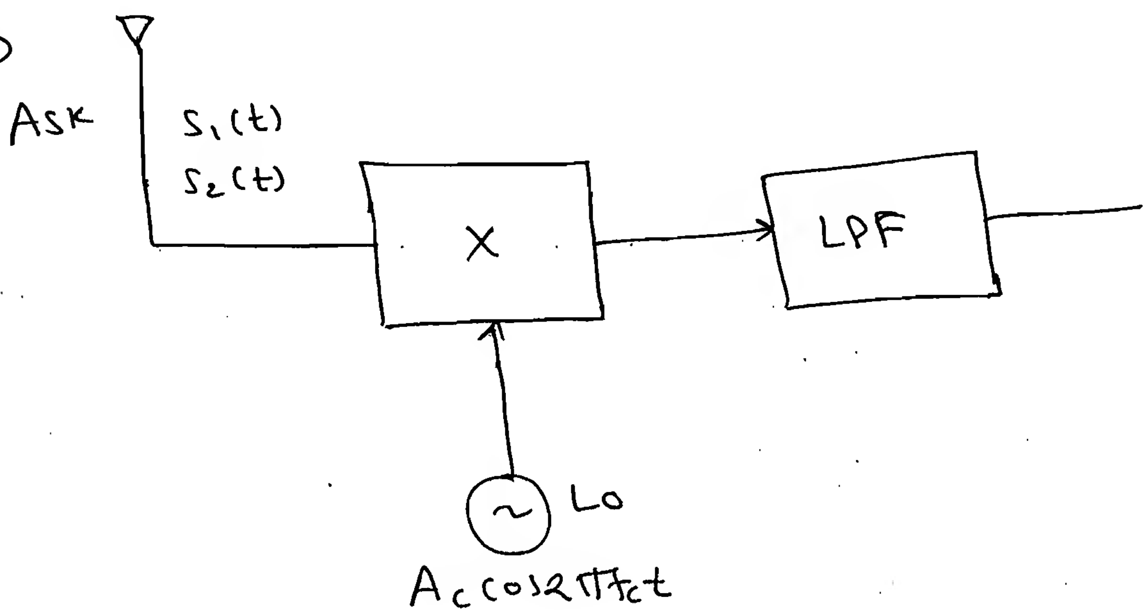


⇒ Mathematical representation of ASK signal is as follow:

$$\boxed{\begin{aligned} S_1(t) &= A_c \cos 2\pi f_c t \quad \text{'1'.} \\ &= 0 \quad \text{'0'.} \end{aligned}}$$

* **Demodulation of ASK:**

⇒



(Coherent | synchroneous detector)

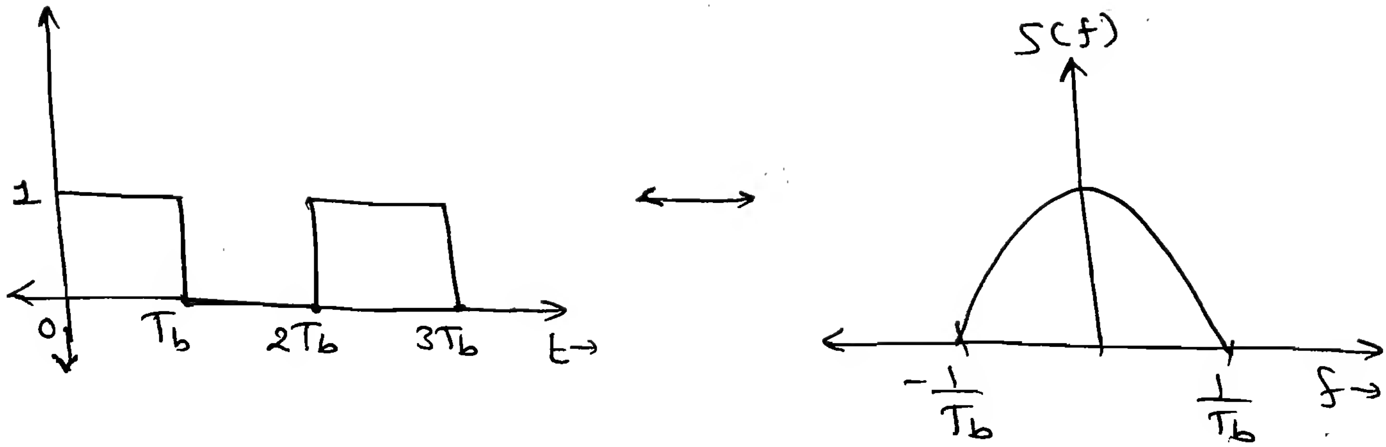
⇒ O/P of the multiplier is

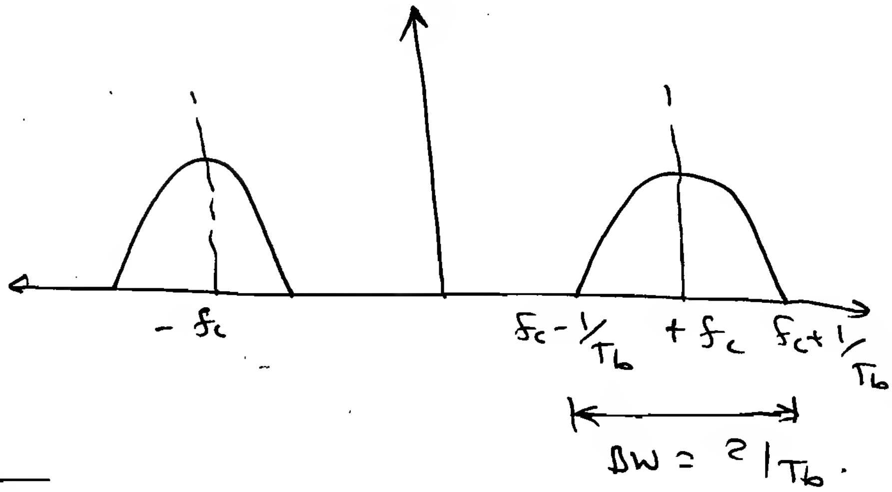
→ $S_1(t) \cdot (Lo)_o = A_c \cos 2\pi f_c t \cdot A_c \cos 2\pi f_c t$

$$= \underbrace{\frac{A_c^2}{2}}_{dc} + \underbrace{\cancel{\frac{A_c^2}{2} \cos 2\pi f_c t}}_{AC}$$

∴ O/P of the LPF = $\frac{A_c^2}{2}$.

$\Rightarrow$





$\Rightarrow$ $BW = 2 \times \frac{1}{T_b}$.

$$\boxed{BW = 2 \times R_b.}$$

* Energy Calculation:

$\Rightarrow$ $E_b = \int_0^{T_b} S_1^2(t)\,dt = P \times T_b.$

$\therefore$ $E_b = \frac{A_c^2}{2} \times T_b.$ '1'.

$= 0$ '0'.

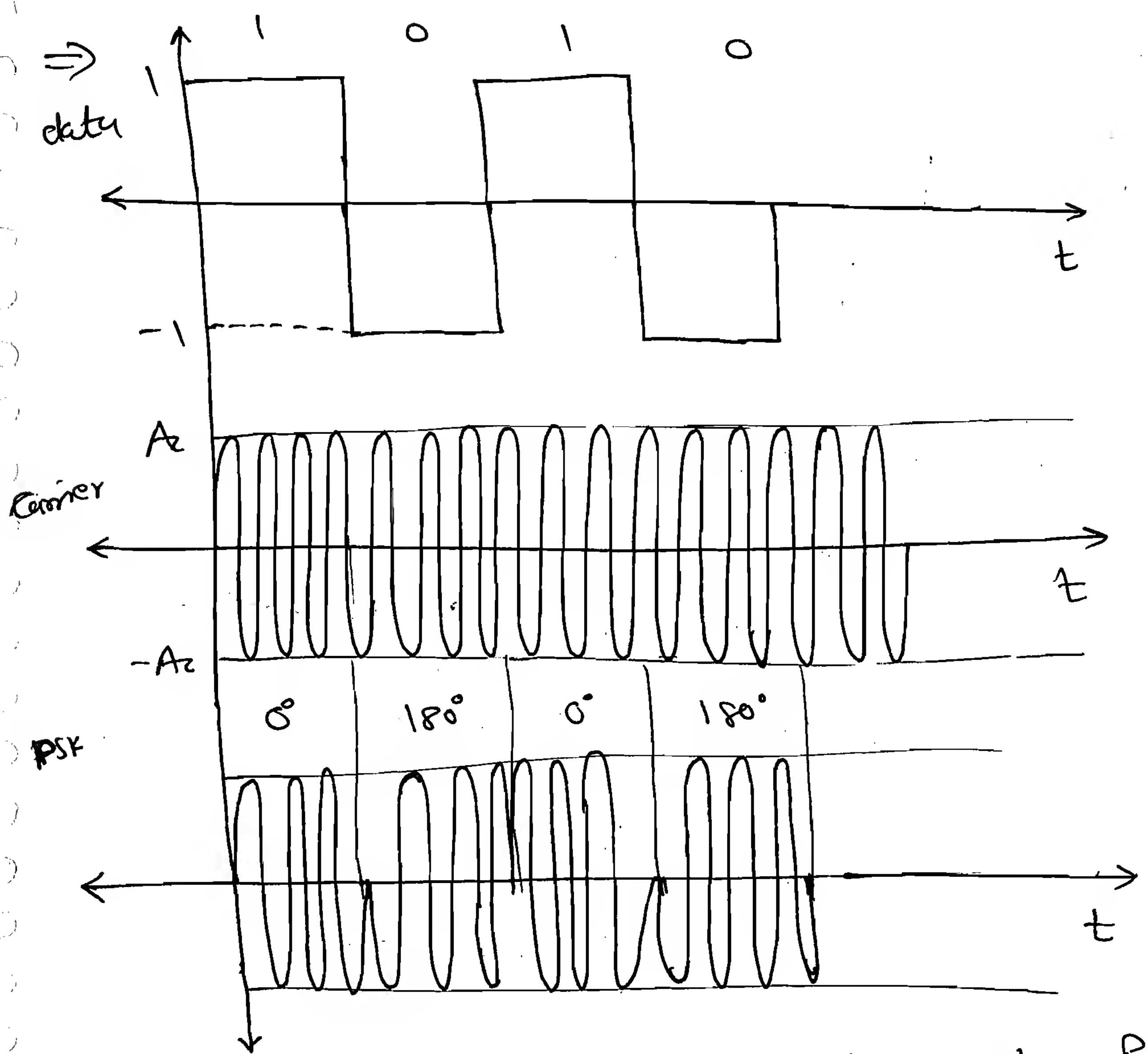
$\Rightarrow$ $\boxed{A_c = \sqrt{\frac{2E_b}{T_b}}}$ $\Rightarrow$ $S_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t$ '1'

$= 0$ '0'.

② PSK (Phase Shift keying):

⇒ Binary data represented in NRZ signalling is multiplied with the carrier to generate the PSK signal.

⇒



⇒ Mathematical Representation of PSK are as follow:

$$\boxed{\begin{aligned} S_1(t) &= A_c \cos 2\pi f_c t & \text{'1'} \\ S_2(t) &= -A_c \cos 2\pi f_c t & \text{'0'} \end{aligned}}$$

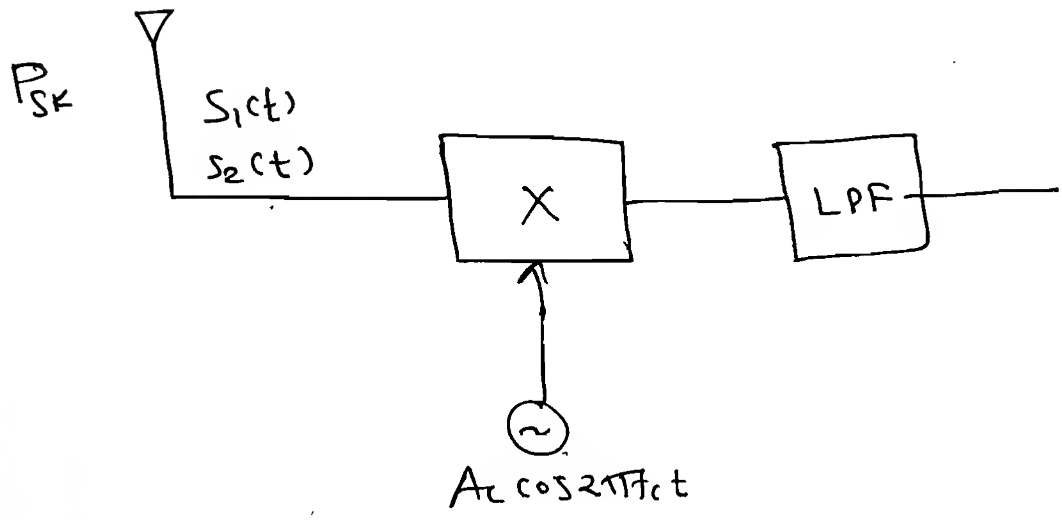
⇒ $S_1(t) = A_c \cos [2\pi f_c t + 0°]$

$S_2(t) = A_c \cos [2\pi f_c t + 180°]$

* Demodulation of PSK:

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cdot \cos 2\pi f_c t$$

$$s_2(t) = -\sqrt{\frac{2E_b}{T_b}} \cdot \cos 2\pi f_c t.$$



⇒ O/P of the multiplier,

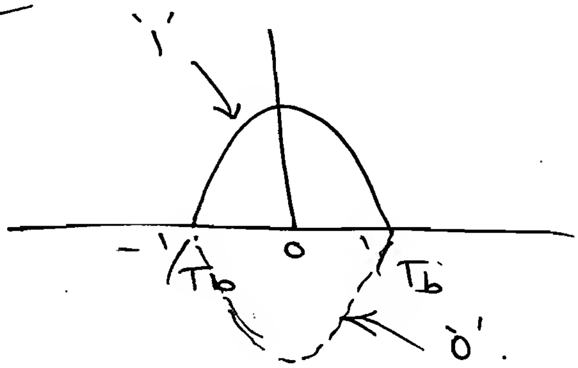
$$s_1(t) \cdot (LO)_o = A_c \cos 2\pi f_c t \cdot A_c \cos 2\pi f_c t$$

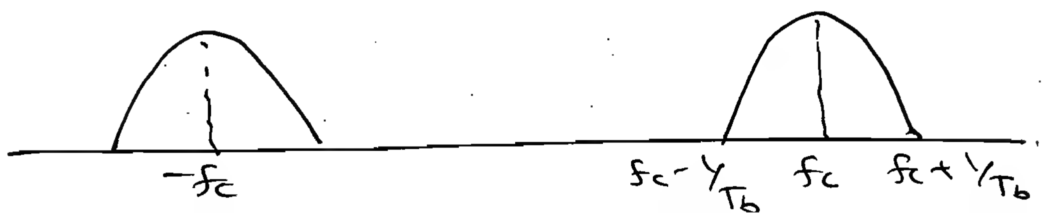
$$= \frac{A_c^2}{2} + \frac{A_c^2}{2} \cdot \cancel{\cos 2\pi (2f_c t)}.$$

$$s_2(t) \cdot (LO)_o = -A_c^2 \cos 2\pi f_c t \cdot \cos 2\pi f_c t$$

$$= -\frac{A_c^2}{2} - \frac{A_c^2}{2} \cancel{\cos 2\pi (2f_c) t}.$$

* Energy:





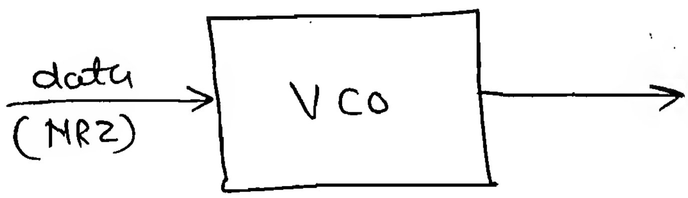
$$\Rightarrow \quad E_b = \int_0^{T_b} s^2(t)\,dt = P \times T_b.$$

$$\therefore \quad E_b = \frac{A_c^2}{2} \cdot T_b. \quad '1' \qquad A_c = \sqrt{\frac{2E_b}{T_b}}.$$

$$E_b = \frac{A_c^2}{2} \cdot T_b \quad '0'.$$

③ Frequency Shift Keying:

$\Rightarrow$



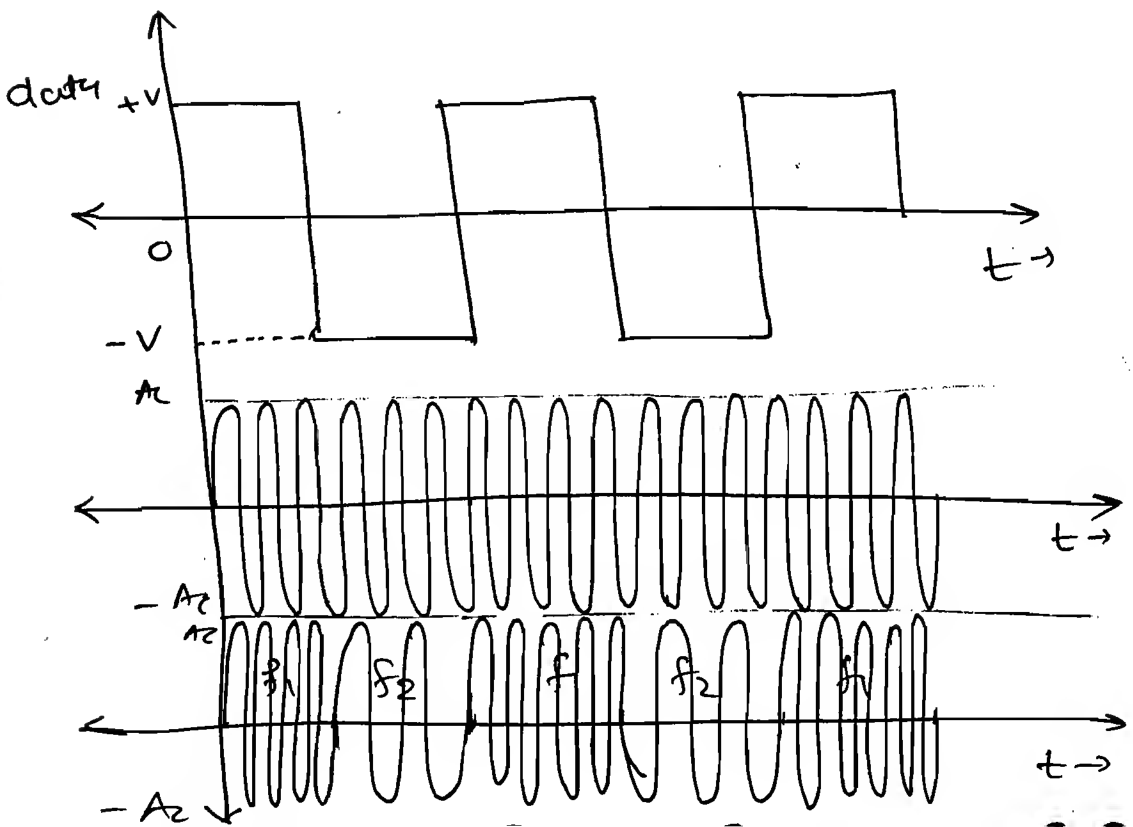
$1 \rightarrow +V \qquad f_1$

$0 \rightarrow -V \qquad f_2$

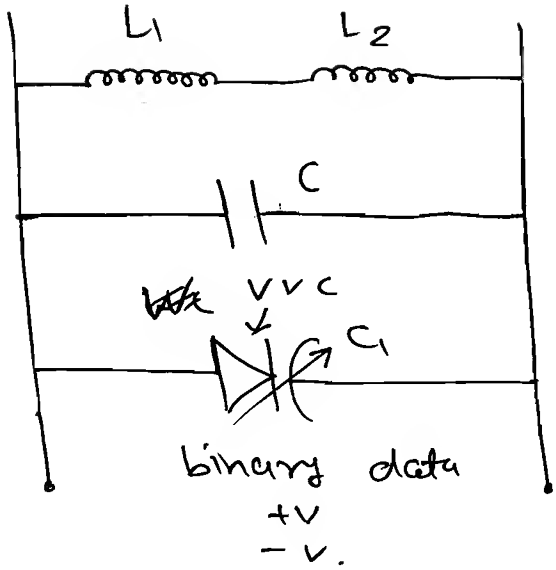
$$f_i = f_c + k_f m(t).$$

$$\therefore \quad f_1 = f_c + k_f \cdot V = \text{Mark freq.}$$

$$f_2 = f_c - k_f \cdot V = \text{Space freq.}$$



⇒



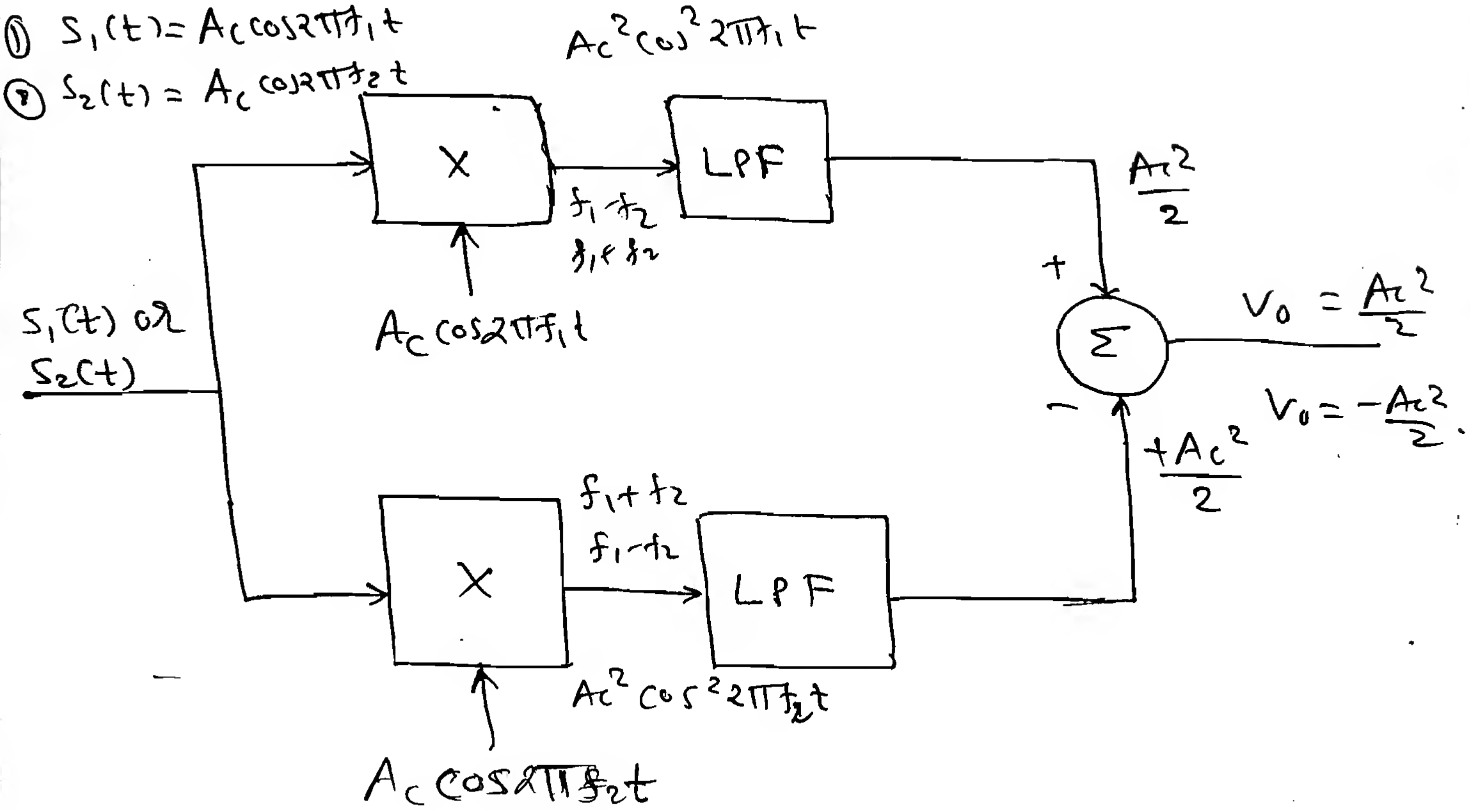
$$\Rightarrow \quad S_1(t) = A_c \cos 2\pi f_1 t.$$

$$S_2(t) = A_c \cos 2\pi f_2 t.$$

* Demodulation of FSK:

① $S_1(t) = A_c \cos 2\pi f_1 t$

② $S_2(t) = A_c \cos 2\pi f_2 t$



$$\Rightarrow \quad E_b = \frac{A_c^2}{2} \cdot T_b. \qquad \text{'1'}$$

$$E_b = \frac{A_c^2}{2} \cdot T_b \qquad \text{'0'}.$$

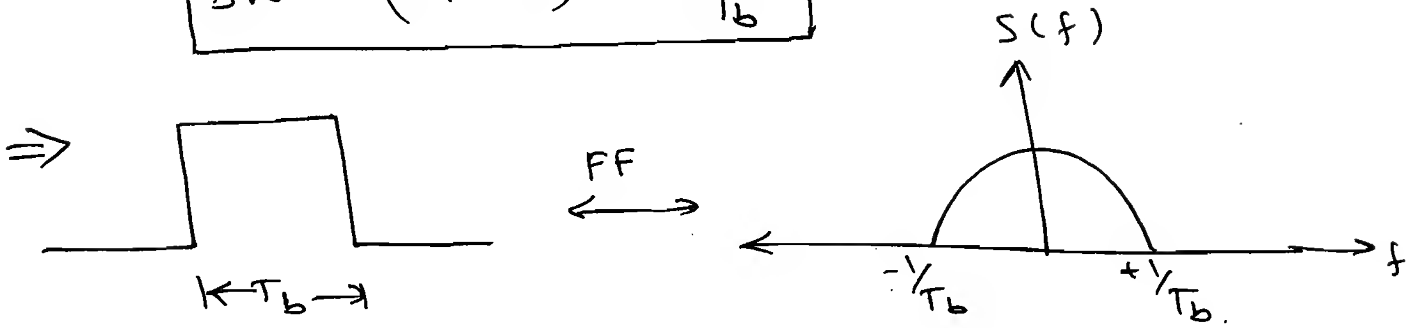
$$\Rightarrow \quad f_{max} = f_c + \Delta f.$$

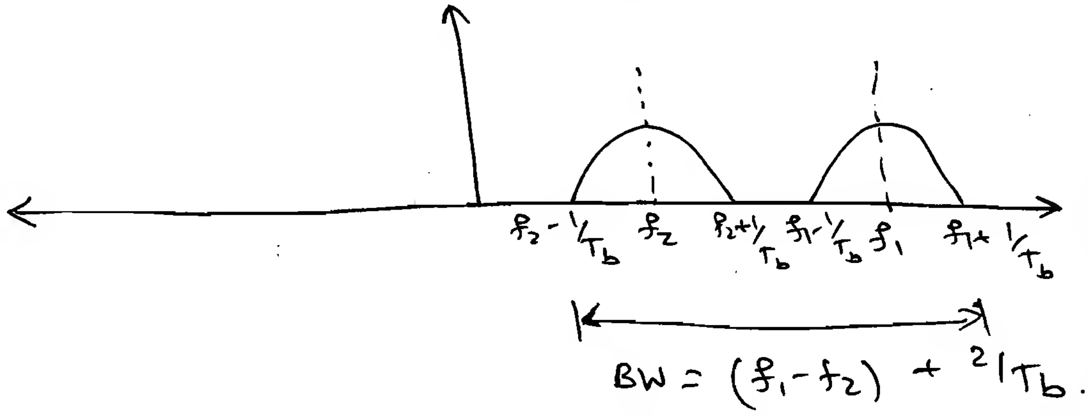
$$f_{min} = f_c - \Delta f.$$

$\Rightarrow$ $BW = 2\Delta f + 2f_m$

$BW = (f_{max} - f_{min}) + 2f_m.$

$\therefore$ $\boxed{BW = (f_1 - f_2) + \frac{2}{T_b}.}$





Q A Voice signal is sampled at the rate of 8000 samples/sec and each sample is encoded into 8-bits using PCM. The binary data is transmitted into free space after modulation. Determine the BW of the modulated signal when the modulation used is

① ASK ② PSK ③ FSK. ($f_1 = 10\,MHz$, $f_2 = 8\,MHz$).

Soln:

Samping rate = 8000 samples / sec.

$n = 8.$

$\therefore$ $R_b = \frac{1}{T_n} \times n.$

$R_b = 8000 \times 8$

$R_b = 64$ kbps.

① ASK:

$$BW = 2R_b$$
$$= 2 \times (64 \text{ kbps})$$
$$\boxed{BW = 128 \text{ KHz}}$$

② PSK:

$$BW = 2R_b.$$
$$BW = 2 \times (64 \text{ kbps})$$
$$\boxed{BW = 128 \text{ kHz}}$$

③ FSK:

$$BW = (f_1 - f_2) + 2R_b.$$
$$= (10-8)\,m + 0.128\,m.$$
$$\boxed{BW = 2.128 \text{ MHz}}$$

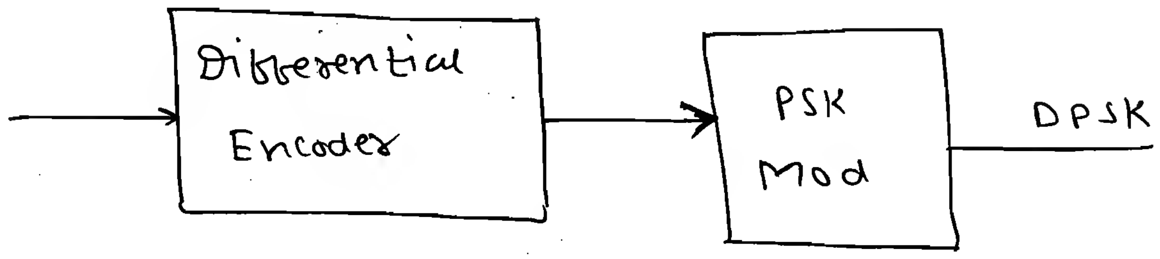
⇒ In FSK BW is very high and in ASK Probability of error is very high.

So, optimum technique is PSK.

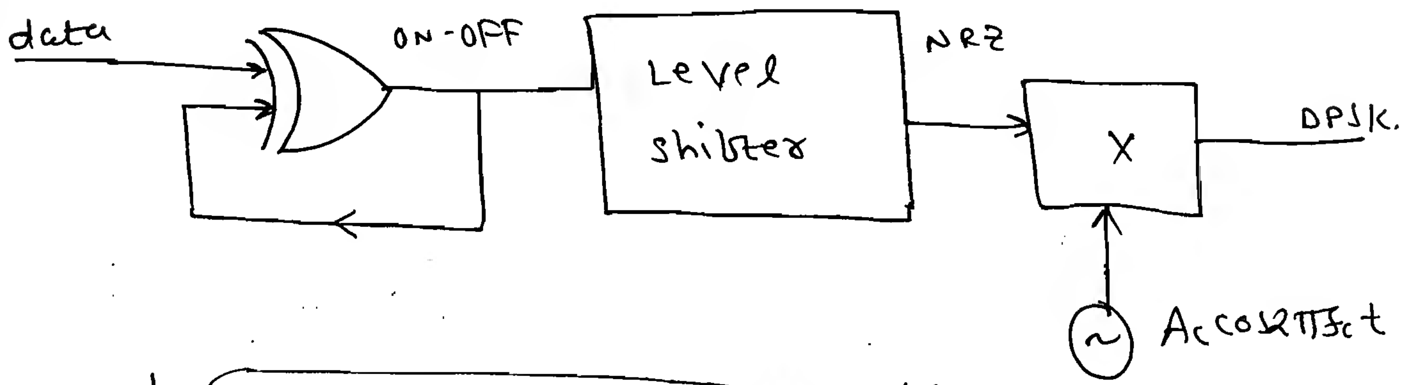
☆ Differential PSK:

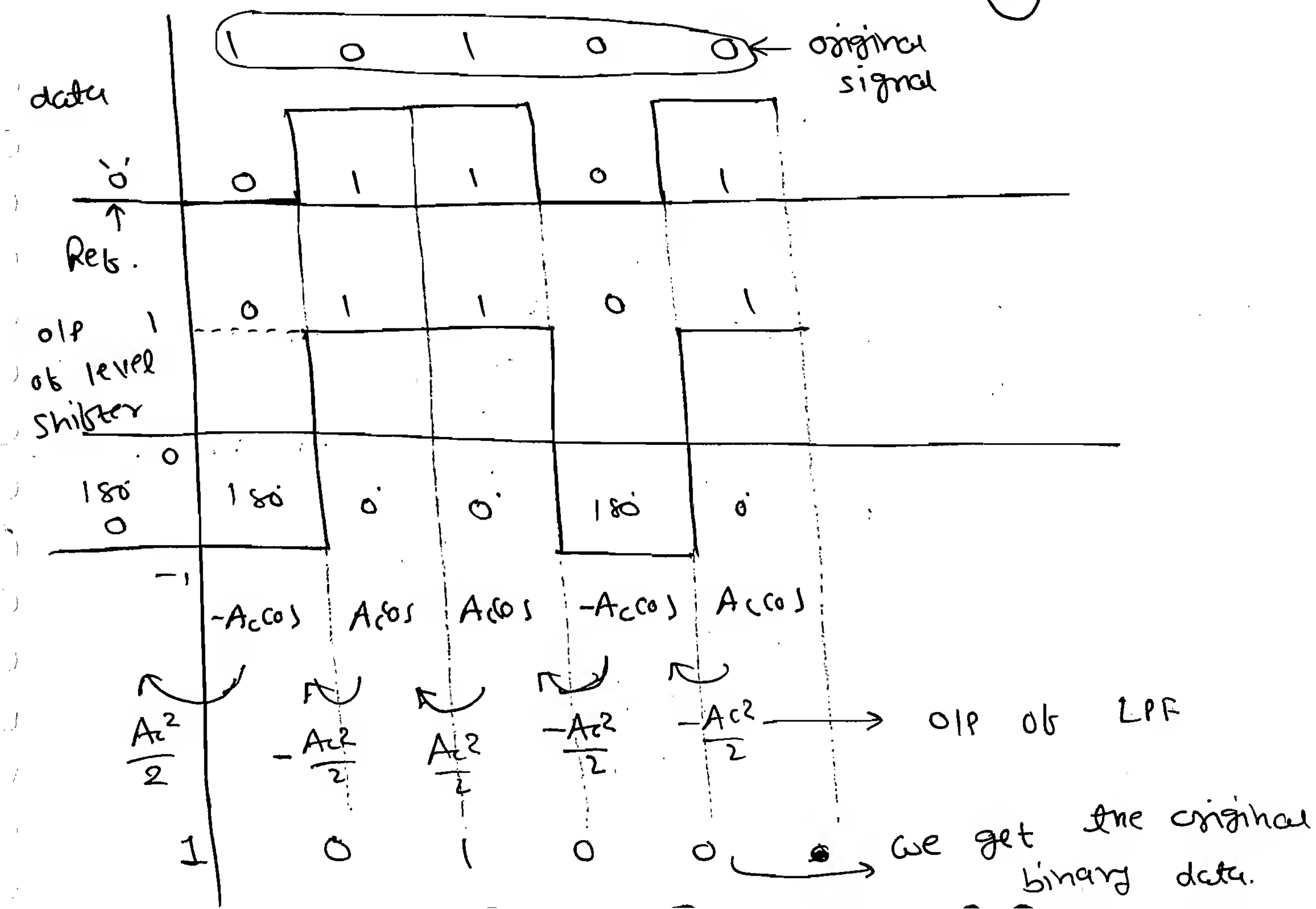
⇒ The binary data is differentially encoded and applied to a PSK modulator to generate the DPSK signal.

⇒



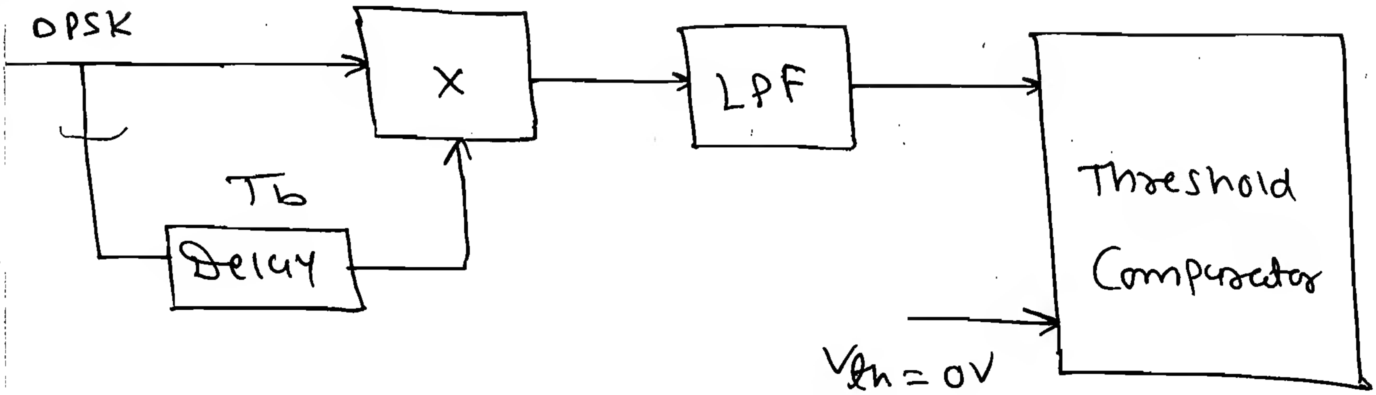
⇒





* Demodulation of DPSK:

⇒



⇒ Local oscillator is not required in DPSK, so, synchronization is not required betⁿ the Tx & Rx.

⇒ 001001   001001

'1' → 011011   '0' → 100100

π 0 0 π 0 0   0 π π 0 π π.

☆ M-ary signaling:

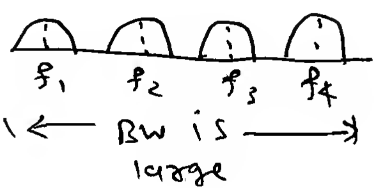
⇒ Binary signaling

M=2 → 1 → f_1 BFSK
0 → f_2

→ 1 → ϕ_1 BPSK
0 → ϕ_2

→ M=4   QFSK

—— f_1
—— f_2
—— f_3
—— f_4



→ M=4   QPSK

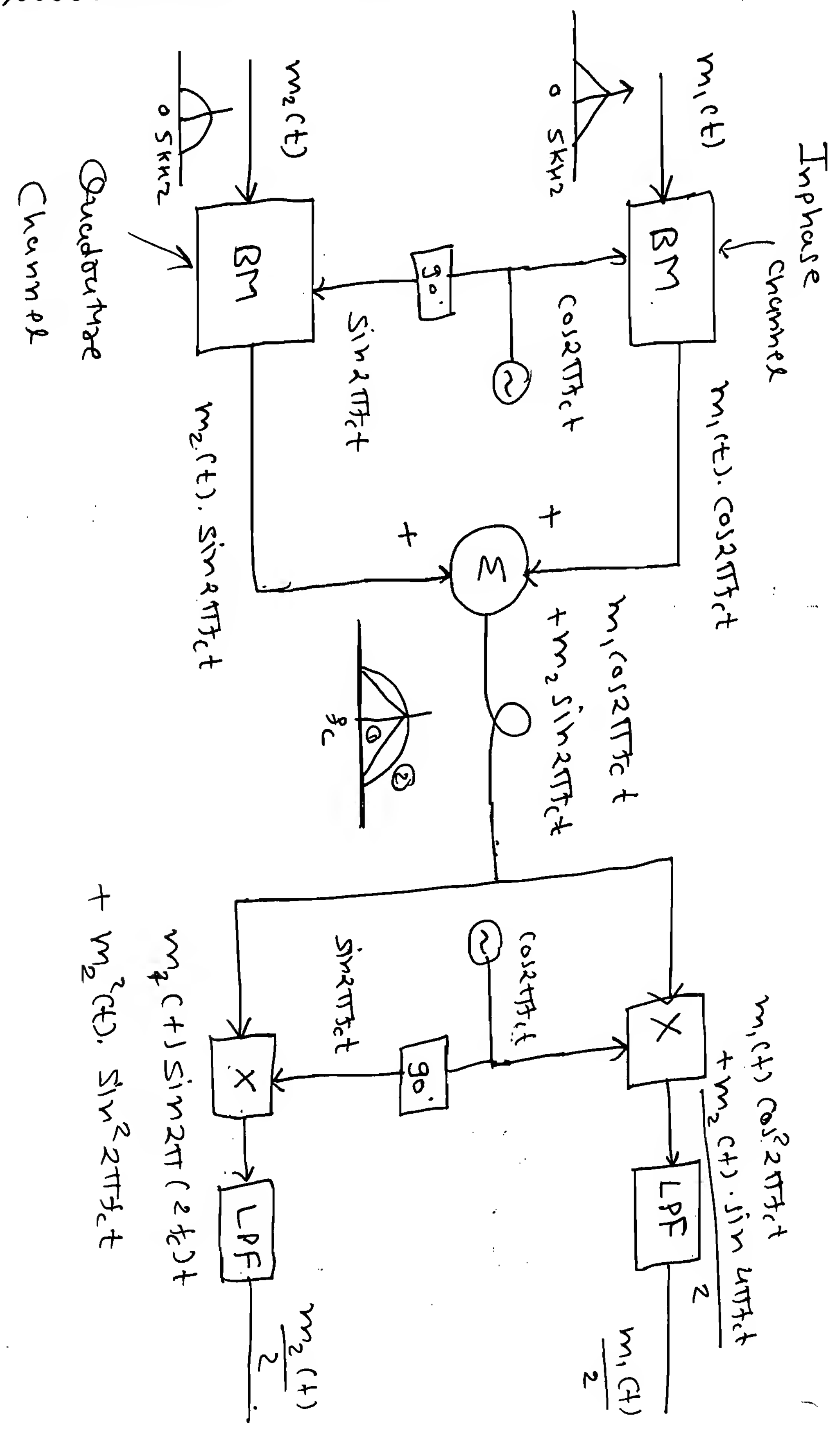
—— ϕ_1
—— ϕ_2
—— ϕ_3
—— ϕ_4

M=8 → 8PSK

—— ϕ_1
—— ϕ_2
⋮
—— ϕ_8

⇒ In M-ary signalling M-descrete voltage levels are transmitted into free space using digital communication.

* Quadrature Carrier Multiplexing:



⇒ It is also called as orthogonal freq. division multiplexing.

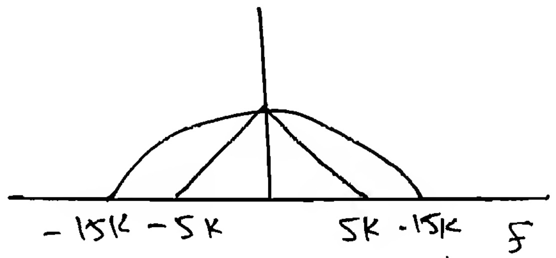
$$\int f_1(t) \cdot f_2(t)\, dt = 0.$$

⇒ It is used in CDMA mobile phone.

Q Consider a multiplexed signal $m_1(t)\cos 2\pi f_c t + m_2(t) \cdot \sin 2\pi f_c t$. $m_1(t)$ is band-limited to 10 KHZ and $m_2(t)$ is band-limited to 15 KHZ. Determine BW of the multiplexed signal:

<u>Soln</u>:

⇒

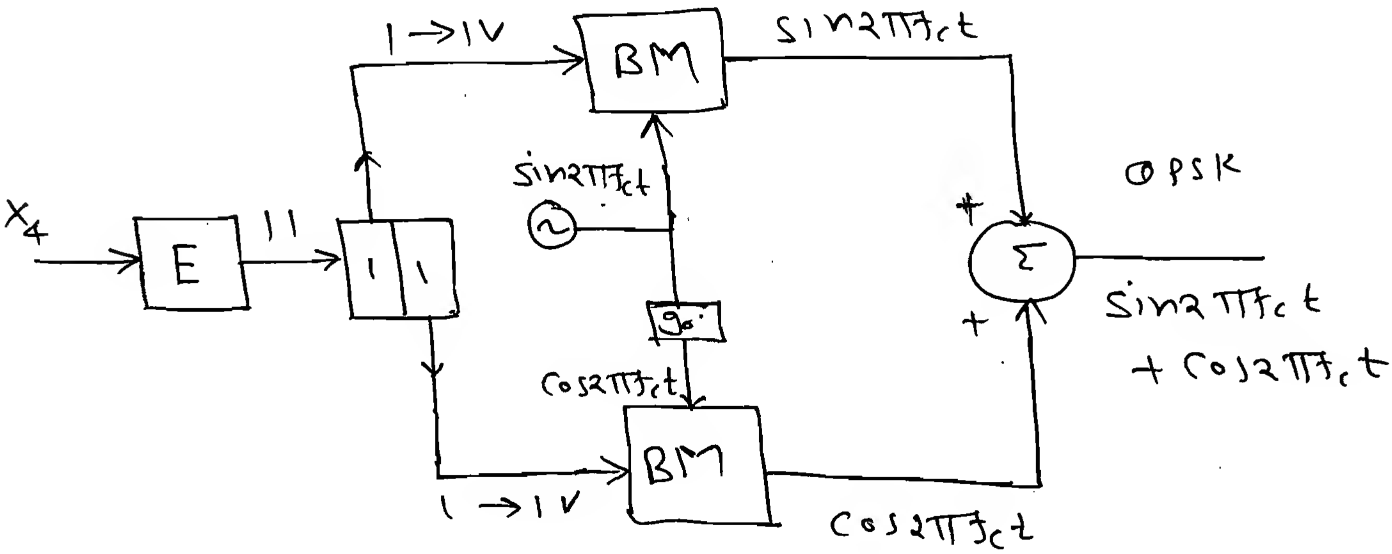


⇒ BW = 2 x highest freq.

$$\boxed{BW = 30\ KHZ}$$

* Quadrature phase shift keying: (QPSK).

⇒



$\Rightarrow$ X_1 —— $00 \rightarrow \phi_1 = -135°$.

X_2 —— $01 \rightarrow \phi_2 = 135°$.

X_3 —— $10 \rightarrow \phi_3 = -45°$.

X_4 —— $11 \rightarrow \phi_4 = 45°$.

$\Rightarrow X_1 \rightarrow 00 \rightarrow -\sin 2\pi f_c t - \cos 2\pi f_c t$

$= \sqrt{2}\,(\sin(2\pi f_c t + 225°))$

$= \sqrt{2}\,(\sin(2\pi f_c t - 135°))$.

$\Rightarrow X_2 \rightarrow 01 \rightarrow -\sin 2\pi f_c t + \cos 2\pi f_c t$

$= \sqrt{2}\,\sin(2\pi f_c t + 135°)$.

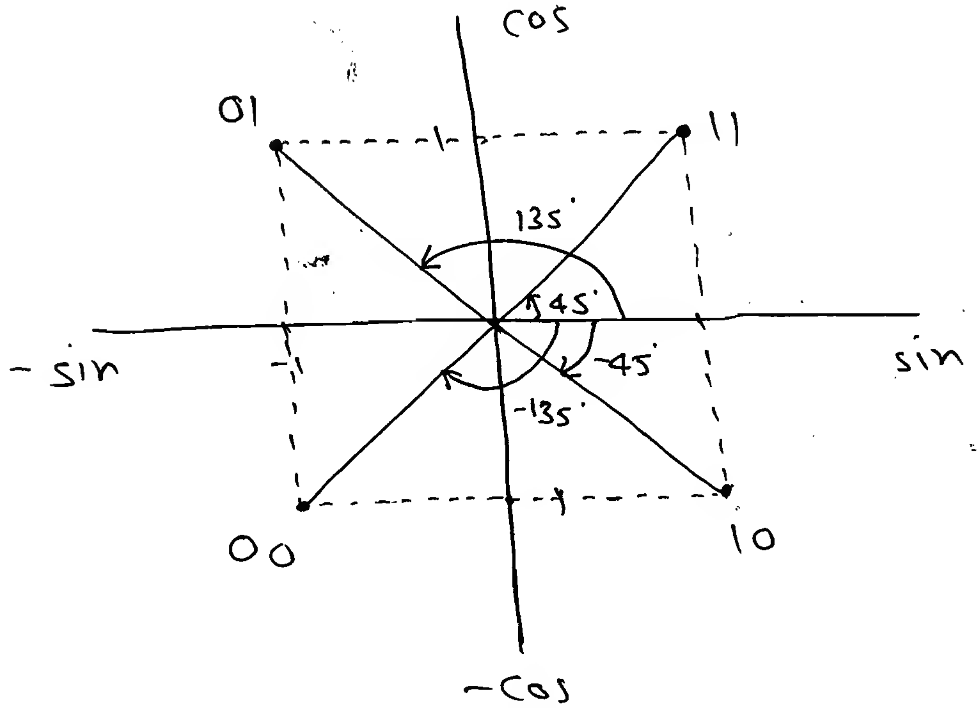
$\Rightarrow X_3 \rightarrow 10 \rightarrow \sin 2\pi f_c t - \cos 2\pi f_c t$

$= \sqrt{2}\,\sin(2\pi f_c t - 45°)$

$\Rightarrow X_4 \rightarrow 11 \rightarrow \sin 2\pi f_c t + \cos 2\pi f_c t$

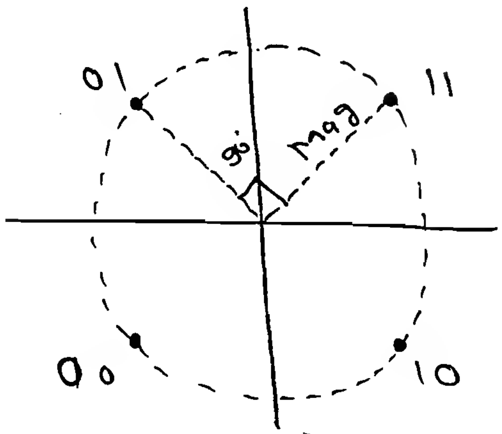
$= \sqrt{2}\,\sin(2\pi f_c t + 45°)$.

* **Phasor Diagram:**

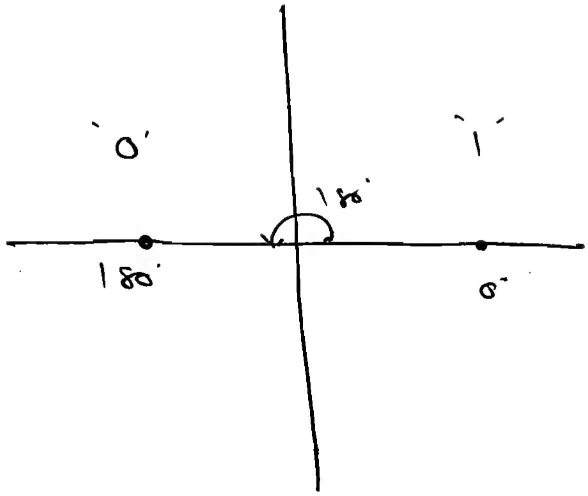


* Constellation Diagram:

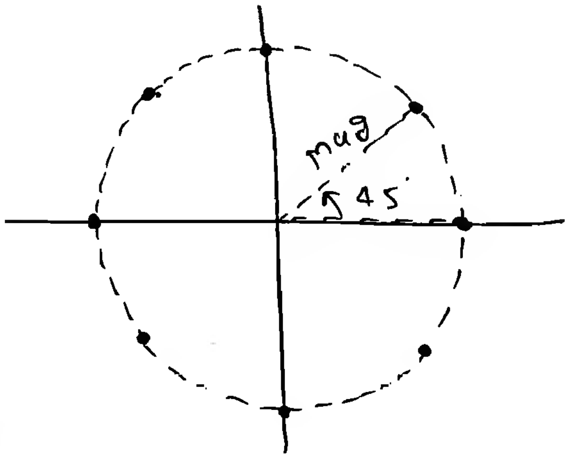
① QPSK:

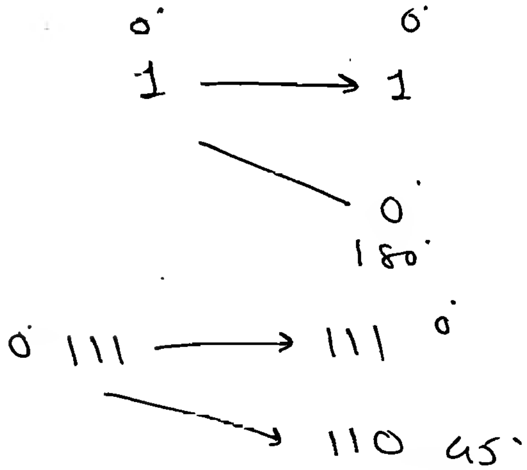


② BPSK:



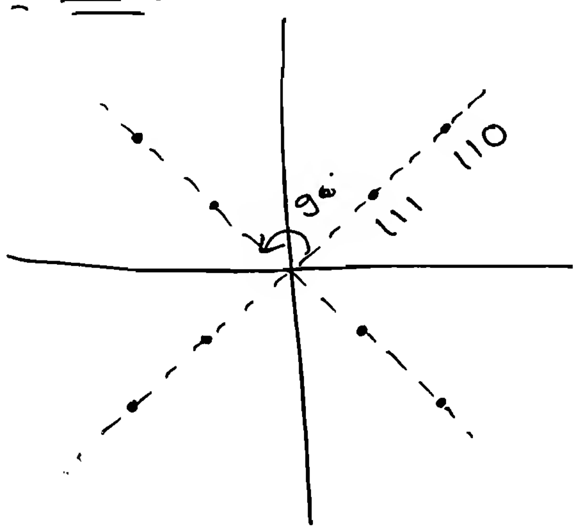
③ 8PSK:





⇒ As M increases angular suppression decreases (or) probability of error increases.

④ 8 QAM

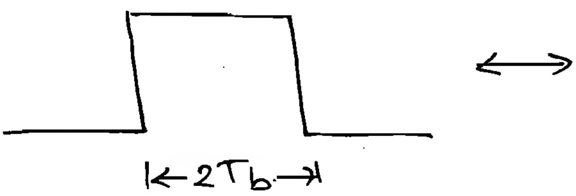


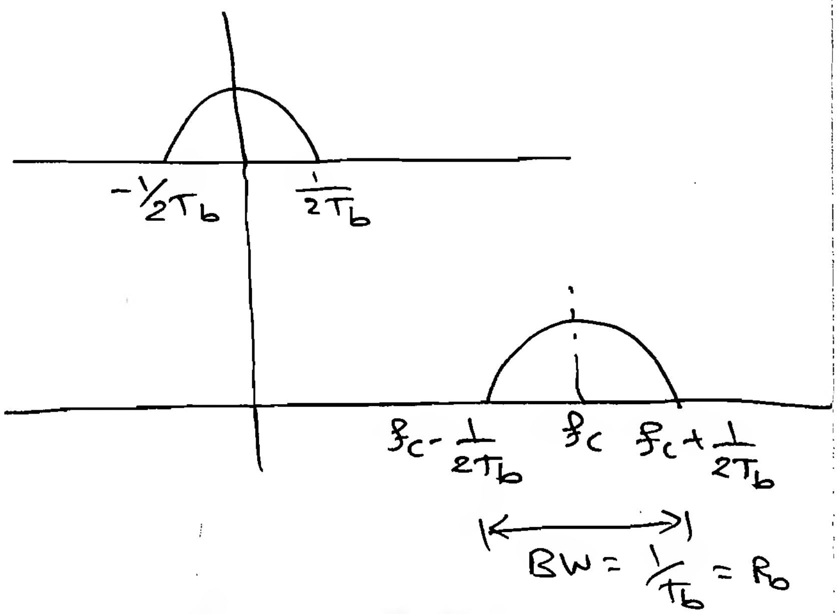
⇒ The input to the QPSK modulator is 2 bits at a time. The duration of this two bits is $2T_b$ sec. one bit is applied to the inphase channel and the other bit is applied to the Quadrature channel. at the same time.

⇒ The i/p to the Balanced Modulator is a rectangular pulse but the pulse width is $2T_b$ sec.

⇒ one pulse is multiplied with the $\cos 2\pi f_c t$ and one pulse is multiplied by $\sin 2\pi f_c t$. So, both occupy the same freq. range but interference not occure as the carriers are orthogonal to each other.

⇒





$$\boxed{BW = \frac{1}{T_b} = R_b}$$

∴ BW = Bit rate

* Generalized formula to determine BW of M-ary signaling:

$$\Rightarrow \quad \boxed{BW = \frac{2R_b}{\log_2 M}} \quad \text{IMP}$$

① BPSK → $M = 2$

$$BW = \frac{2R_b}{\log_2 2} = 2R_b$$

② QPSK (QAM) → $M = 4$

$$BW = \frac{2R_b}{\log_2 2^2} = \frac{2R_b}{2\log_2 2} = R_b.$$

③ 8-PSK → $M = 8$

$$BW = \frac{2R_b}{\log_2 2^3} = \frac{2}{3}R_b.$$

* Band rate:

Bit rate → Band rate

110110 → [BPSK] → 0° 0° 180° 0° 0° 180°

6 bits / sec → 6 phase / sec

110110 → [QPSK] → 45° 135° −45°

6 bits / sec → 3 phase / sec.

110110 → [8PSK] → 2 phase / sec.

6 bits / sec

$$\Rightarrow \boxed{\text{Baud rate} = \frac{\text{Bit rate}}{\log_2 M} = \frac{R_b}{\log_2 M} = \frac{BW}{2}}$$

$\Rightarrow$ Band width efficiency :-

$$\boxed{\eta = \frac{R_b}{BW}}$$

$$BPSK \rightarrow \eta = \frac{R_b}{2R_b} = 50\%$$

$$QPSK \rightarrow \eta = \frac{R_b}{2R_b} = 100\%$$

☆ Noise Analysis of Digital Communication:

* Quantization Noise:

$\Rightarrow$ In PCM system input to the LPF is the sampled signals with quantization error. Due to this error in each sample signal distortion occurs and this distortion is called as the quantization noise. So, the quantization error should be as minimum as possible.

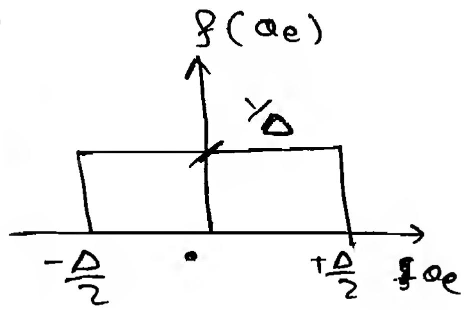
$\Rightarrow$ Quantization error depends on the step size.

⇒ Assume that the input is a sinusoidal signal

$$m(t) = A_m \cdot \cos 2\pi f_m t.$$

$$\& \quad P = \frac{A_m^2}{2} \ (W).$$

⇒ The noise occur due to the quantization error. So, the mean square value of the quantization error represents the quantization noise power.

⇒ 

Quantization noise power

$$= \int Q_e^2 \cdot f(Q_e) \, dQ_e.$$

∴ Quantization noise power

$$= \int_{-\frac{\Delta}{2}}^{+\frac{\Delta}{2}} Q_e^2 \cdot \left(\frac{1}{\Delta}\right)^2 \cdot dQ_e.$$

$$= \frac{2}{\Delta^2} \times \left[\frac{Q_e^3}{3}\right]_0^{\frac{\Delta}{2}}.$$

$$= \frac{2}{\Delta^2} \times \frac{\Delta^3}{48 \times 3}$$

$$= \frac{\Delta^2}{12}$$

$\therefore$ $\boxed{\text{Quantization noise power} = \frac{\Delta^2}{12}}$

$$\Rightarrow \quad SQNR = \frac{P_{si}}{P_{QN}} = \frac{\frac{A_m^2}{2}}{\frac{\Delta^2}{12}} = \frac{6 A_m^2}{\Delta^2 \times \cancel{8}}.$$

$$\text{But} \quad \Delta = \frac{V_{p-p}}{L} = \frac{2 A_m}{L}.$$

$$\therefore \quad SQNR = \frac{\cancel{A_m^2}}{4 \cancel{A_m^2}} \times \frac{L^2}{\cancel{8}} \times 6$$

$$SQNR = 3/_2 \, L^2$$

$$L = 2^n.$$

$$\therefore \quad \boxed{SQNR = \frac{3}{2} \times 2^{2n}.}$$

IMP

$$\boxed{\begin{aligned} &(SQNR)_{dB} = (\alpha + 6n)\ dB. \\ &\alpha = 1.8 \ \text{for uncompressed} \\ &\alpha = 10 \log_{10} C \ \text{for Compressed.} \\ &\rightarrow C = \frac{3}{[\ln[1+\mu]]^2} \end{aligned}}$$

$$\therefore \quad (SQNR)_{dB} = 10 \log \left[\frac{3}{2} \times 2^{2n} \right]. = 10 \log 3/2 + 10 \log 2^{2n}$$

$$= 1.76 + 20 n \log 2$$

$$= 1.76 + 6n.$$

$$\therefore \quad \boxed{(SQNR)_{dB} = (1.8 + 6n)\ dB}$$

$\Rightarrow$ $n = 2 \Rightarrow 13.8$ dB. } 6 dB / bit.

$n = 3 \rightarrow 19.8$ dB. } 6 dB / bit.

$n = 4 \rightarrow 25.8$ dB.

IMP

$\Rightarrow$ As n increases, the Δ decreases and the quantization noise power also decreases. So, the signal to noise ratio increases and the improvement in the SNR is $\boxed{6\text{ dB per bit.}}$

Q In a PCM system the Code length is increased from 6 to 8 bits. The Quantization power is decreases by a factor of (A) 2 (B) 2 (C) 8 (D) 16.

Soln:

$n=6 \Rightarrow \Delta \Rightarrow P_{qn} = \frac{\Delta^2}{12}$.

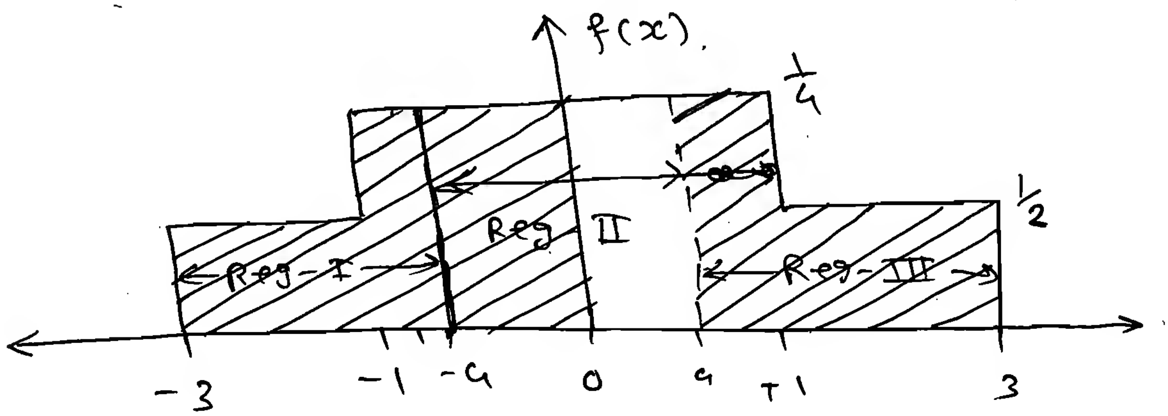
$n=7 \Rightarrow \frac{\Delta}{2} \Rightarrow P_{qn} = \frac{\Delta^2}{4\times 12}$.

$n=8 \Rightarrow \frac{\Delta}{4} \Rightarrow P_{qn} = \frac{\Delta^2}{16\times 12}$.

So, ans is (D) 16

Q Gate - 2005 Linked Type Question

A 3 level Quantizer is design assuming the equiprobable occurance of all the 3 quantization levels. The pdf is divided into 3 region as shown in fig.



Q-1 Determine the value of of the a in the above figure.

Q-2 Determine P_{qn} betn $-a$ to $+a$.

Soln: ① ~~A~~ Total area = 1.

$\Rightarrow$ All three region i.e Reg- I, Reg- II and Reg - III are equiprobable.

So, Area of each region is $1/3$.

$\therefore$ Area of Reg- II $= 1/3$.

$$\therefore \quad 2a \times \frac{1}{\cancel{4}\,2} = \frac{1}{3}$$

$$\therefore \quad \boxed{a = 2/3}$$

②

$$P_{on} = \int_{-a}^{a} x^2 . f(x). dx$$

$$= \int_{-2/3}^{2/3} \frac{1}{4} . x^2 . dx .$$

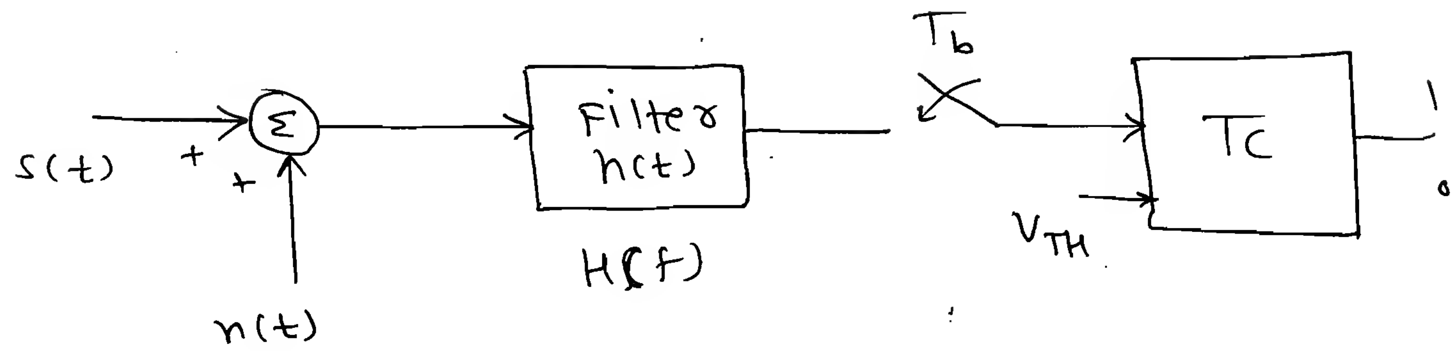
$$= \frac{2}{4} \times \left[\frac{x^3}{3} \right]_0^{2/3} .$$

$$= \frac{1}{\cancel{2}} \times \frac{\cancel{8}\, 4}{3 \times 9 \times 3}$$

$$\boxed{P_{on} = \frac{4}{81} \text{ W.}}$$

☆ Matched Filter Receiver :-

=>



=> The received signal and noise is applied to the filter whose impulse response is $h(t)$. The output of the filter is sample at every T_b sec. The sampled value is compared with the Threshold voltage to take decision weither is '0' (or) '1'. If the received signal is very large then effect of the noise is very large then binary symbol '1' will be received as '0' and '0' will be received as '1'. This error should be as minimum as possible. So, the SNR should be as high as possible.

=> The signal at the o/p of the filter is,

$$S_0(t) = s(t) \otimes h(t).$$

$$\Rightarrow S_0(f) = S(f) \cdot H(f).$$

$$\therefore \quad S_0(t) = \int_{-\infty}^{\infty} S(f) \cdot H(f) \cdot e^{j2\pi f t} \cdot df.$$

$\Rightarrow$ Assume that the o/p signal is sample at $t = T_b$ sec.

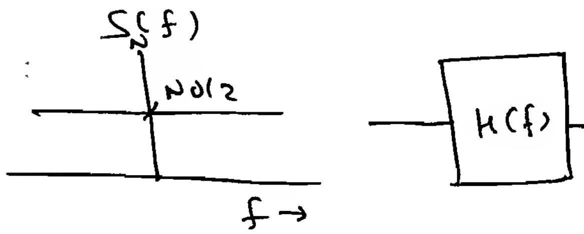
so, $$S_0(T_b) = \int_{-\infty}^{\infty} S(f) \cdot H(f) \cdot e^{j2\pi f T_b} \cdot df.$$

$\Rightarrow$ The signal power at the threshold comparator is,

$$\text{signal power} = |S_0(T_b)|^2$$

$$= \left| \int_{-\infty}^{\infty} S(f) \cdot H(f) \cdot e^{j2\pi f T_b} \cdot df \right|^2.$$

$\Rightarrow$



$$(PSD)_0 = \frac{N_0}{2} |H(f)|^2 \quad W/Hz.$$

$$\text{Power} = \int_{-\infty}^{\infty} (PSD)_0 = \frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 \cdot df.$$

$$\Rightarrow (S/N) = \frac{\left| \int_{-\infty}^{\infty} H(f) \cdot S(f) \cdot e^{j2\pi f T_b} \right|^2}{\frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 \cdot df}$$

* **Schwartz inequality:**

$$\left| \int_{-\infty}^{\infty} f_1(t) \cdot f_2(t) \, dt \right|^2 \leq \int_{-\infty}^{\infty} |f_1(t)|^2 dt \cdot \int_{-\infty}^{\infty} |f_2(t)|^2 \cdot dt.$$

LHS = RHS

only when $f_1(t) = f_2^*(t)$.

$$\therefore (S/N) \leq \frac{\cancel{\int_{-\infty}^{\infty} |H(f)|^2 \cdot df} \cdot \int_{-\infty}^{\infty} \left| S(f) \cdot e^{j2\pi f T_b} \right|^2 \cdot df}{\frac{N_0}{2} \cancel{\int_{-\infty}^{\infty} |H(f)|^2 \cdot df}}$$

$$\therefore (S/N) \leq \frac{\int_{-\infty}^{\infty} |S(f)|^2 \cdot df}{\frac{N_0}{2}} \qquad \left(\because \left| e^{j2\pi f T_b} \right| = 1 \right).$$

$s(t) \rightarrow S(f)$

$|S(f)|^2 \rightarrow$ ESD

$\int_{-\infty}^{\infty} |S(f)|^2 df \rightarrow E$

$$\therefore (S/N) \leq \frac{E}{(N_0/2)}.$$

H.B

IMP

$$\therefore \boxed{(S/N)_{max} = \frac{2E}{N_0}}.$$

only when $H(f) = S^*(f) \cdot e^{-j2\pi f T_b}$.

$$\Rightarrow h(t) = \int_{-\infty}^{\infty} H(f) \cdot e^{j2\pi f t} \cdot df.$$

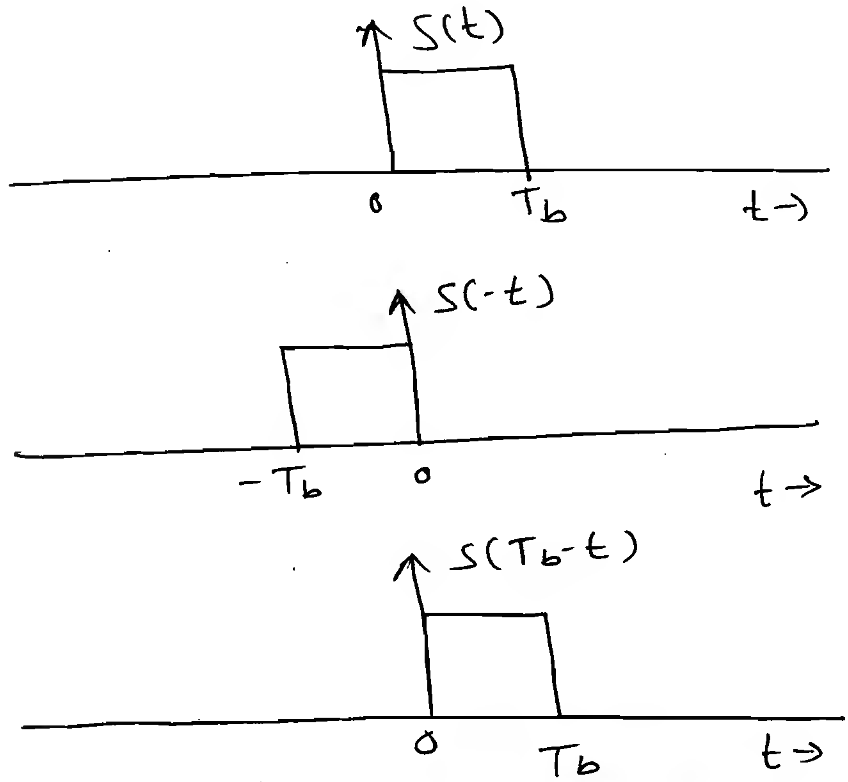
$$\therefore h(t) = \int_{-\infty}^{\infty} S^*(f) \cdot e^{-j2\pi f T_b} \cdot e^{+j2\pi f t} \cdot df$$

$$\therefore h(t) = \int_{-\infty}^{\infty} S^*(f) \cdot e^{-j2\pi f (T_b - t)} \cdot df.$$

$$h(t) = \left[\int_{-\infty}^{\infty} S(f) \cdot e^{j2\pi f (T_b - t)} \cdot df \right]^*$$

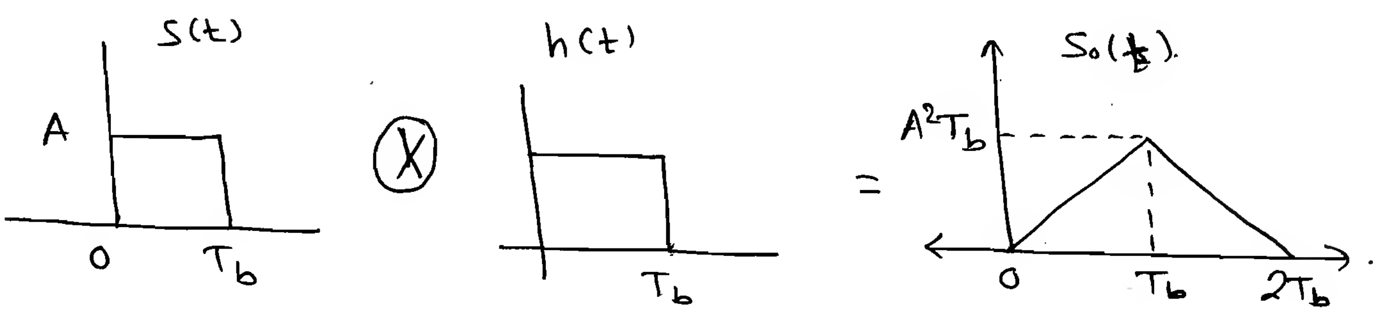
$$\therefore \boxed{h(t) = S^*(T_b - t).}$$

$\Rightarrow$ To determine the impulse response of the filter following procedure is used:



⇒ The impulse response of the filter is same as the ~~impulse~~ input signal (or) impulse response of the filter is matched to i/p signal. ~~Therefore~~ That's why it is called as Matched filter receiver.

⇒ The output of the filter is,



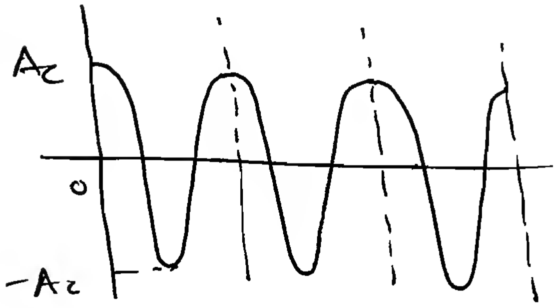
⇒ The Sampled value (or) the max. value is always equal to the energy of signal,

$$E = \int_0^{T_b} A^2 dt.$$

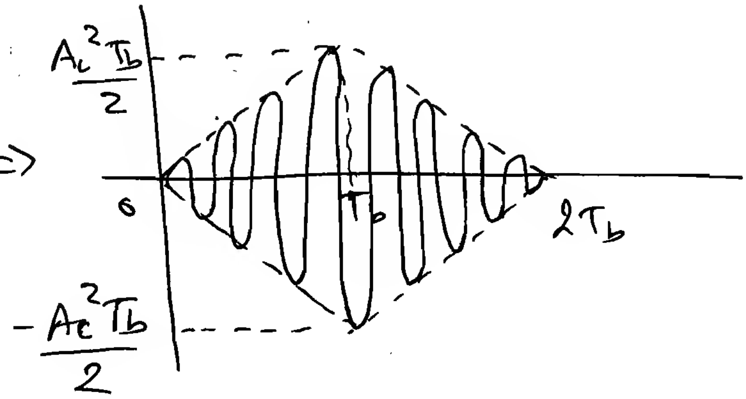
⇒ The input to the threshold Comparator is either A^2T_b (or) 0 when ON - OFF signaling is used.

⇒ The input to the Threshold Comparator is A^2T_b (or) $-A^2T_b$ when NRZ signaling is used.

⇒ Assume that the i/p to the matched filter is $A_c \cos 2\pi f_c t$ where, $T_b = N/f_c$ where N is integer.

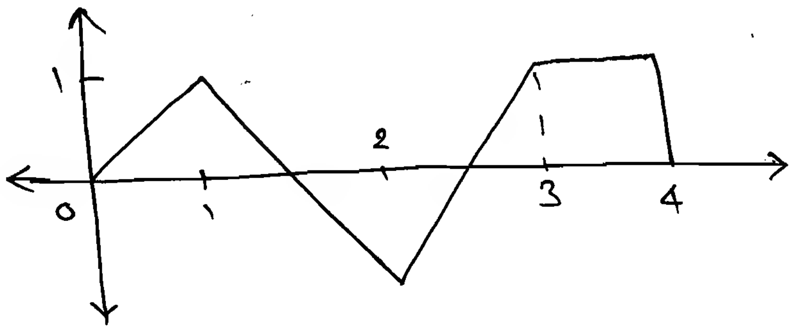


⇒



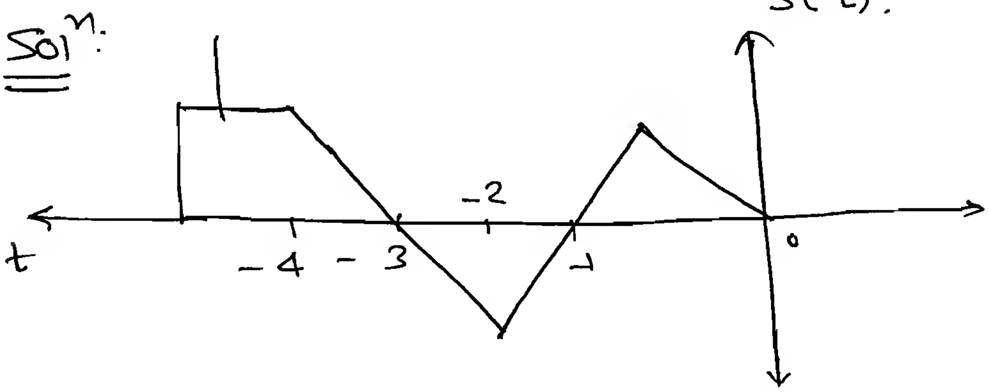
⇒ The impulse response of the filter will be matched to the input signal only when Rectangular pulse and carrier.

$\boxed{Q}$ The i/p to the matched filter is
Gate-2004
as shown in the figure.

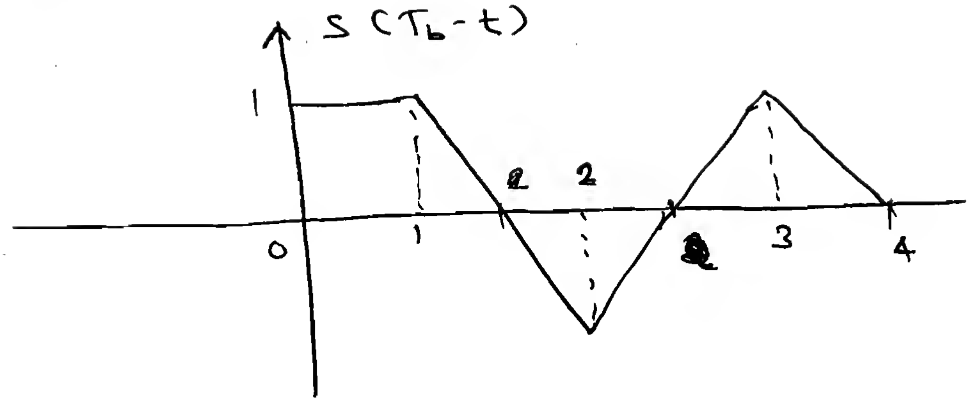


Determine the slope of h(t) in the interval 3 to 4.

Sol^n:

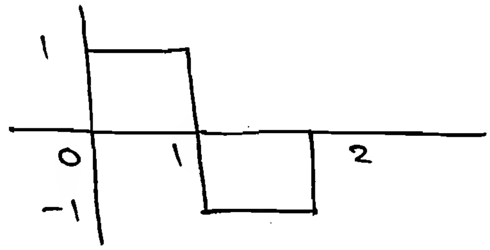


⇒

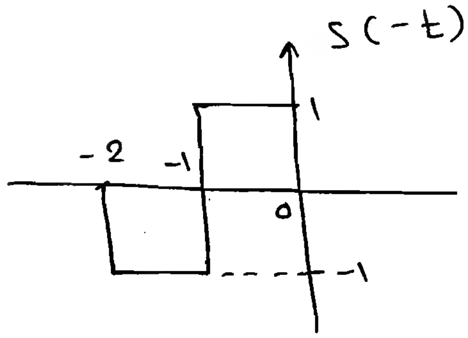


$$\text{Slope} = \frac{1-0}{3-4} = -1$$

$\boxed{Q-2}$ The i/p to the matched filter is as shown in figure. Determine the h(t).

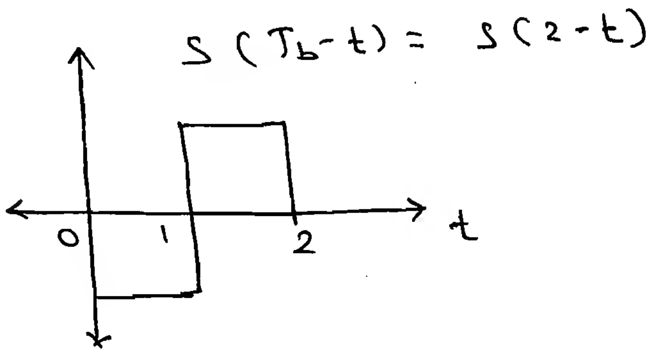


Soln:



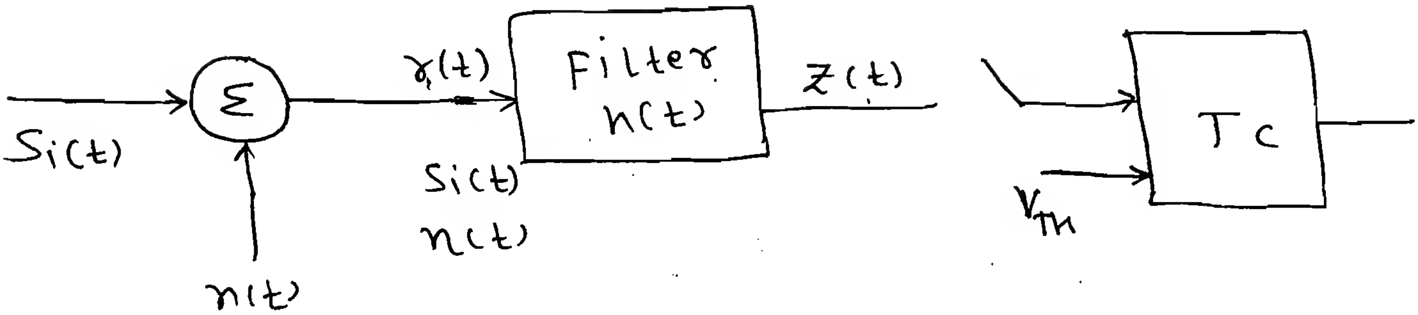
⇒

⇓



☆ Probability of Error Calculation: (IMP)

⇒



⇒ $r(t) = S_i(t) + n(t).$

$z(t) = a_i(t) + n_o(t).$

→ assume that the signal is sampled at $t = T_b$.

$\therefore z(T_b) = a_i(T_b) + n_o(T_b).$

→ The input to the Threshold comparator

$z = a_i + n_o.$

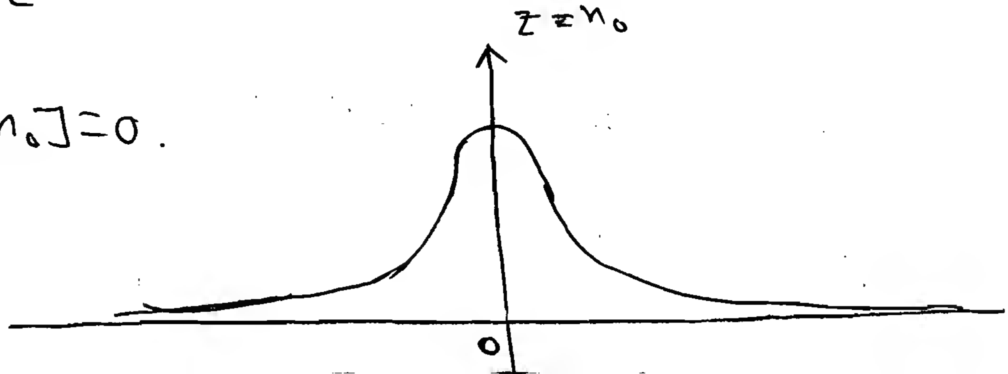
Case - I:

⇒ Assume that the signal is not present at the input to the receiver.

$z = n_o$

⇒ n_o is assumed as a gaussian random variable with zero mean.

$E[z] = E[n_o] = 0.$



Case- II:

⇒ Assume that the binary symbol 1 is present at the input of the receiver.

⇒ $$Z = a_1 + n_0$$

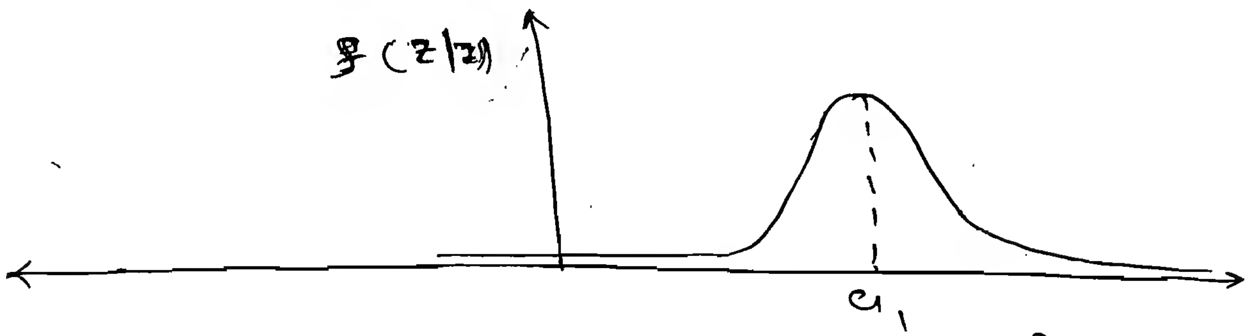
$$E[z] = E[a_1 + n_0]$$

$$= E[a_1] + E[n_0]$$

$$E[z] = a_1 + 0$$

$$\therefore \boxed{E[z] = a_1}$$

⇒ In this case also z is Gaussian random variable with mean $= a_1$.



$$\therefore f(z/1) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(z-a_1)^2}{2\sigma^2}}$$

Case - III:

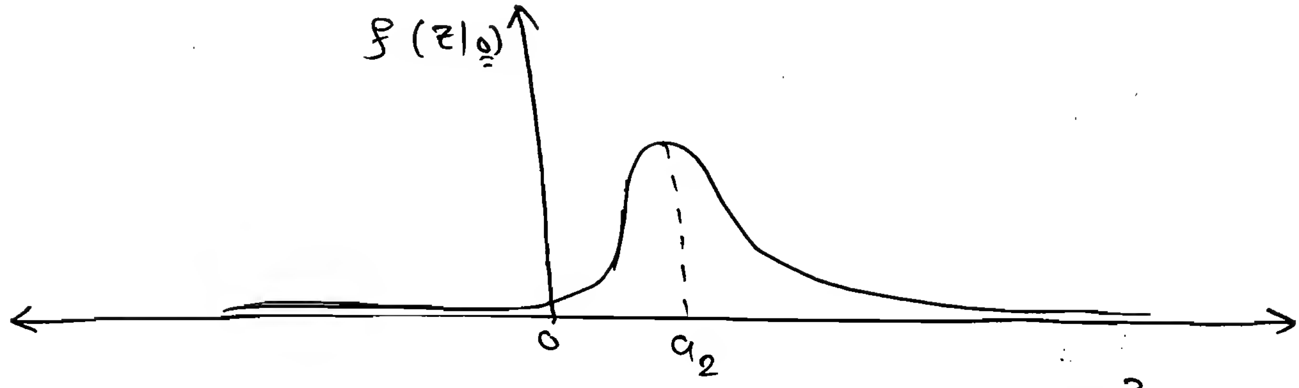
⇒ Assume that the binary signal 0 is present at the input to the receiver.

⇒ $$Z = a_2 + n_0.$$

$$\therefore E[Z] = E[a_2] + E[n_0]$$

$$\therefore \boxed{E[z] = a_2}$$

$\Rightarrow$



$\Rightarrow \quad f(z|0) = \dfrac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(z-a_2)^2}{2\sigma^2}}$.

$\Rightarrow$ The i/p to the Threshold comparator is the avg. value of a_1+n_0 & a_2+n_0.

$$\text{So, } \quad z = \frac{a_1+a_2}{2} = V_{th}.$$

$\Rightarrow$ Assume that the binary symbol 1 is transmitted through the channel.

$\Rightarrow$ The i/p to the threshold comparator $z = a_1+n_0$. The o/p of the comparator will be '0' (or) error occurs if the following condition is satisfied:

$\Rightarrow \quad 1 \longrightarrow$ error $[z < V_{th}]$.

$$\boxed{P_{e_1} = P[z < V_{Th}].} \leftarrow \underline{\text{H.B.}}$$

similarly.

$0 \longrightarrow$ error occurs $[z > V_{TH}]$.

$$\boxed{P_{e_0} = P[z > V_{TH}].} \leftarrow \underline{\text{H.B.}}$$

$\Rightarrow$ $$\boxed{P_{e_1} = \int_{-\infty}^{V_{TH}} f(z|1)\, dz} \quad -①$$ $\leftarrow$ H.B.

$$\boxed{P_{e_0} = \int_{V_{TH}}^{\infty} f(z|0)\, dz} \quad -②$$ $\leftarrow$ H.B.

$\Rightarrow$ Practically the effect of noise on the binary symbol '0' and '1' will be same. So, the prob. of error will also be same. i.e. $P_{e_1} = P_{e_0}$.

$\Rightarrow$ $$P_e = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{\frac{a_1+a_2}{2}}^{\infty} \cdot e^{\frac{(z-a_2)^2}{2\sigma^2}} \cdot dz.$$

$\Rightarrow$ The above integral can be represent in a function.

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-y^2/2} \cdot dy.$$

~~let,~~ Property : $Q(x) \downarrow$ when $x \uparrow$

let, $y = \frac{z-a_2}{\sigma} \Rightarrow \frac{dy}{dz} = \frac{1}{\sigma} \Rightarrow \boxed{dz = \sigma\, dy}$

$$y = \frac{\frac{a_1+a_2}{2} - a_2}{\sigma} \Rightarrow y = \frac{a_1 - a_2}{2\sigma}.$$

$$\therefore \quad P_e = \frac{\sigma}{\sqrt{2\pi\sigma^2}} \int_{\frac{a_1-a_2}{2\sigma}}^{\infty} e^{-y^2/2}\, dy$$

$$\therefore \quad \boxed{P_e = Q\left[\frac{a_1-a_2}{2\sigma}\right].}$$

$$\therefore \quad P_e = Q\left[\sqrt{\frac{(a_1-a_2)^2}{4\sigma^2}}\right].$$

mean square value = power = σ^2.
= Noise power.

$$\therefore \quad P_e = Q\left[\sqrt{\frac{\text{Difference signal power}}{4 \times \text{Noise power}}}\right].$$

$$\therefore \quad P_e = Q\left[\sqrt{\frac{2E_d}{4N_0}}\right].$$

$$\therefore \quad \boxed{P_e = Q\left[\sqrt{\frac{E_d}{2N_0}}\right].}$$

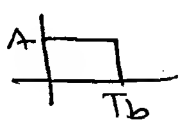
H.B.
MIMP.

$$\therefore \quad \boxed{E_d = \int_0^{T_b} [s_1(t) - s_2(t)]^2\, dt}$$

* Probability of Error in PCM System:

$\Rightarrow$ ON-OFF

$s_1(t) = A$



$s_2(t) = 0.$

NRZ

$s_1(t) = A$

$s_2(t) = -A.$

$\therefore\ S_1(t) - S_2(t) = A$

$$\therefore\ E_D = \int_0^{T_b} A^2 \cdot dt$$

$$\therefore\ \boxed{E_D = A^2 \cdot T_b}$$

$$\therefore\ \boxed{P_e = Q\left[\sqrt{\frac{A^2 T_b}{2N_0}}\right]} \leftarrow \text{H.B.}$$

$\uparrow$
X_1

$S_1(t) - S_2(t)$
$= A - (-A) = 2A$

$$\therefore\ E_D = \int_0^{T_b} 4A^2 \cdot dt$$

$$\therefore\ \boxed{E_D = 4A^2 T_b}$$

$$P_e = Q\left[\sqrt{\frac{4A^2 T_b}{2N_0}}\right]$$

$$\boxed{P_e = Q\left[\sqrt{\frac{2A^2 T_b}{N_0}}\right]}$$

$\uparrow$
X_2

H.B.

$X_2 > X_1$

$\therefore\ P_e(X_2) < P(X_1)$.

$\therefore\ \boxed{P_e(\text{NRZ}) < P_e[\text{ON-OFF}]}$.

Q-1 A received NRZ signal assume the voltage level 1 500 mV and −500mV respectively for '1' and '0'. The received signal is affected by white noise having a two sided spectrum density of 10^{-10} W/Hz. Determine the bit rate so that $P_e = 10^{-5}$.

$Q(x) = 10^{-5}$, $x = 4.27$.

Soln:

$A = 500$ mV

$\frac{N_0}{2} = 10^{-10}$ W/Hz $\Rightarrow$ $N_0 = 2 \times 10^{-10}$ W/Hz.

$$\therefore \quad P_e = Q\left[\sqrt{\frac{2A^2T_b}{N_o}}\right] = 10^{-5}.$$

$$\therefore \quad \sqrt{\frac{2A^2T_b}{N_o}} = (4.27)$$

$$\therefore \quad \frac{2A^2T_b}{N_o} = (4.27)^2$$

$$\therefore \quad T_b = \frac{(4.27)^2 \times \cancel{2} \times 10^{-10}}{0.25 \times \cancel{2}}$$

$$\therefore \quad T_b = \underset{72.93}{\cancel{145.8632}} \times 10^{-10}.$$

$$\therefore \quad R_b = \frac{1}{T_b}$$

$$\therefore \quad R_b = \underset{}{\overset{13.71}{\cancel{6.856}}} \times 10^{7}$$

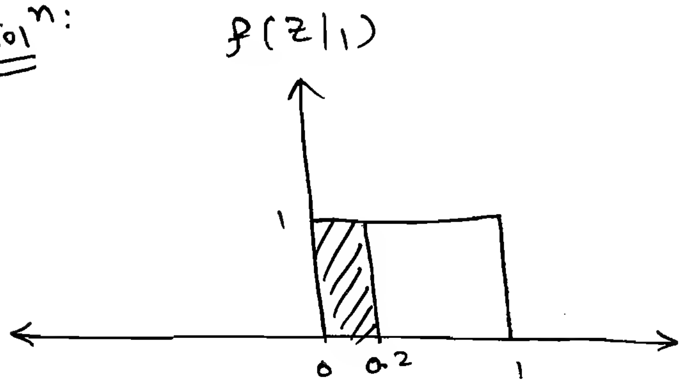
$$\therefore \quad \boxed{R_b = \underset{137}{\cancel{68}} \text{ Mbps.}}$$

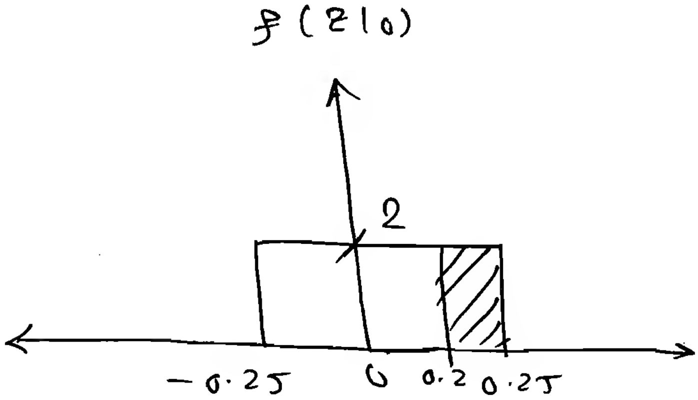
⇒ Probability of error should be minimum only either A (or) T_b should be as maximum (or) very high. So, the bit rate should be as minimum as possible.

$\boxed{Q-2}$ Consider a digital Communication System when the binary symbol 1 is transmitted the input to the Threshold Comparator can be any value betn 0 to 1 Volt. with equal prob.. when binary symbol 0 is transmitted the input to the Threshold Comparator can be any value betn

$-0.25V$ to $+0.25V$ with equal probability. The Threshold value used at the Comparator is $0.2V$. Determine the prob. of 0 & 1. Also calculate the avg. prob. of error.

Soln:





$\therefore$ $1 \rightarrow$ error occurs $[z < 0.2]$.

$$Pe_1 = P[z < 0.2] = \int_{-\infty}^{0.2} f(z|1)\, dz$$

$$\therefore Pe_1 = \int_{0}^{0.2} 1 \cdot dz$$

$$\boxed{Pe_1 = 0.2}$$

$\therefore$ $0 \longrightarrow$ error occurs $[z > 0.2]$.

$$Pe_0 = P[z > 0.2] = \int_{0.2}^{\infty} f(z|0)\, dz$$

$$= \int_{0.2}^{0.25} 2 \cdot dz$$

$$= 2[0.05].$$

$$\therefore \boxed{Pe_0 = 0.1}$$

$$\therefore P_{avg} = \frac{Pe_0 + Pe_1}{2}$$

$$\therefore P_{avg} = \frac{0.1 + 0.2}{2}$$

$$\therefore \boxed{P_{avg} = 0.15}$$

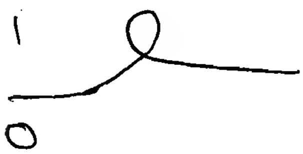
⇒ Prob. of error (Bit error rate) is depends on the threshold value.

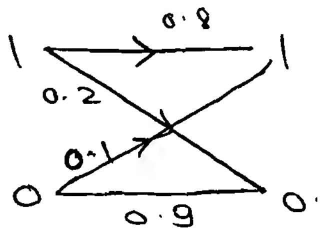
⇒ The threshold value should be selected so that the prob. of error is minimum.

⇒ The optimum threshold value will be equal to the intersection of the two pdf of $\underline{f(z|1)}$ and $\underline{f(z|0)}$. H.B.

* To determine the avg. Probability of error the following procedure is used:

⇒





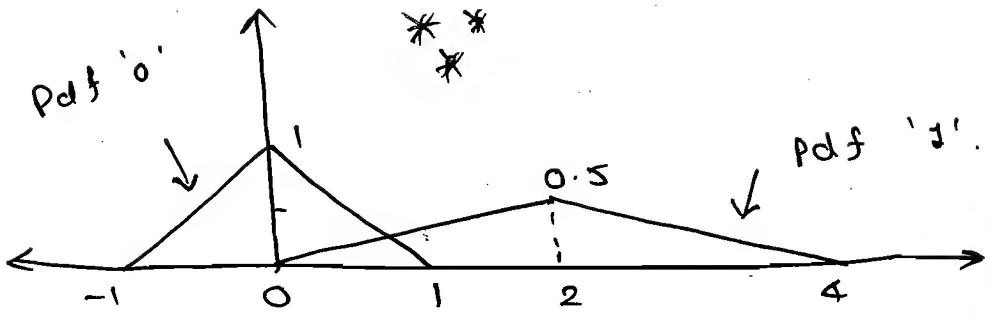
⇒ $$\boxed{P_e = P(1) \cdot P_{e1} + P(0) \cdot P_{e0}}$$

∴ if $P(1) = P(0)$.

∴ $P_e = \frac{1}{2}(0.2) + \frac{1}{2}(0.1)$

$$\boxed{P_e = 0.15}$$

Q Bits 1 and 0 are transmitted with equal probability at the receiver the pdf of the respective received signals are as shown in the figure.



$\boxed{Q\text{-}1}$ If the decision threshold is 1 the bit error rate will be

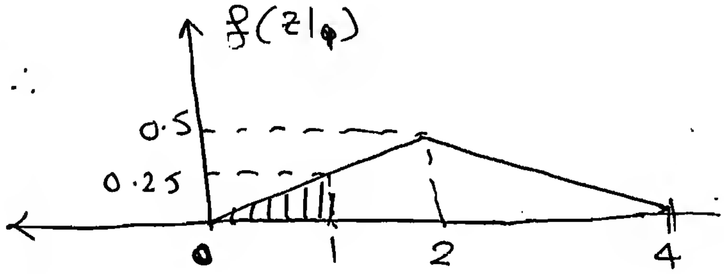
(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{8}$ (D) $\frac{1}{16}$.

<u>Soln</u>:

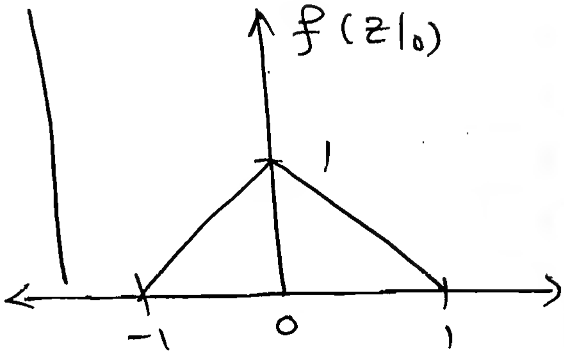
$$P_e = P(1) \cdot P_{e1} + P(0) \cdot P_{e0}.$$

$$P(1) = P(0) = \frac{1}{2}.$$

∴



∴



$$P_{e1} = \int_{-\infty}^{1} f(z|1) \cdot dz$$

$$= \frac{1}{2} \times 1 \times 0.25$$

$$\therefore \boxed{P_{e1} = \frac{1}{8}}$$

$$P_{e0} = \int_{1}^{\infty} f(z|0)\, dz$$

$$\boxed{P_{e0} = 0}$$

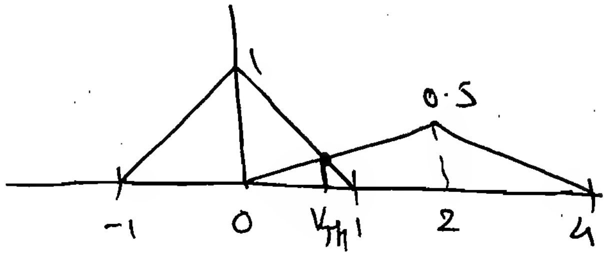
$$\therefore P_e = \left(\frac{1}{2} \times \frac{1}{8}\right) + \left(\frac{1}{2} \times 0\right)$$

$$\boxed{P_e = \frac{1}{16}}$$

Q-2 The optimum threshold to achieve the minimum bit error is ?

(A) $\frac{1}{2}$ (B) $\frac{4}{5}$ (C) 1 (D) $3/2$.

Sol^n:



① $x + y = 1$.

$y = \frac{x}{4}$.

$\Rightarrow x + \frac{x}{4} = 1$

$\therefore x = \frac{4}{5}$.

$\therefore \boxed{V_{Th} = \frac{4}{5} V}$

E_b for ASK = E_b for message signal

*

ASK	PSK	FSK
$\Rightarrow S_1(t) = A_c \cos 2\pi f_c t$	$S_1(t) = A_c \cos 2\pi f_c t$	$S_1(t) = A_c \cos 2\pi f_1 t$
$S_2(t) = 0$	$S_2(t) = -A_c \cos 2\pi f_c t$	$S_2(t) = A_c \cos 2\pi f_2 t$
$E_d = \int_0^{T_b} (A_c \cos 2\pi f_c t)^2 dt$	$E_d = \int_0^{T_b} (2A_c \cos 2\pi f_c t)^2 dt$	$\boxed{E_d = \oint A_c^2 T_b.}$
$\therefore \boxed{E_b = \frac{A_c^2 T_b}{2}}$	$\boxed{E_d = 2A_c^2 T_b.}$	
$\therefore \boxed{P_e = Q\left[\sqrt{\frac{A_c^2 T_b}{4N_0}}\right]}$	$\boxed{P_e = Q\left[\sqrt{\frac{A_c^2 T_b}{N_0}}\right]}$	$\boxed{P_e = Q\left[\sqrt{\frac{A_c^2 T_b}{2N_0}}\right]}$
$\boxed{BW = 2R_b}$	$\boxed{BW = 2R_b}$	$\boxed{BW = f_1 - f_2 + 2R_b}$

$x_2 > x_3 > x_4$

So, $P(x_2) < P(x_3) < P(x_4)$.

F.R.

⇒ The Probability of error for ASK is high when Compared with FSK and PSK. The BW of the FSK signal is high when Compared with ASK and PSK. So, the optimum technique in digital Communication is PSK.

⇒ $E_b = \frac{A_c^2 T_b}{2}$

H.B

ASK

$\therefore P_e = Q\left[\sqrt{\frac{E_b}{2N_o}}\right]$

PSK

$P_e = Q\left[\sqrt{\frac{2E_b}{N_o}}\right]$

FSK

$P_e = Q\left[\sqrt{\frac{E_b}{N_o}}\right]$.

$P_e = Q\left[\sqrt{\frac{E_b \cos^2\phi}{2N_o}}\right]$

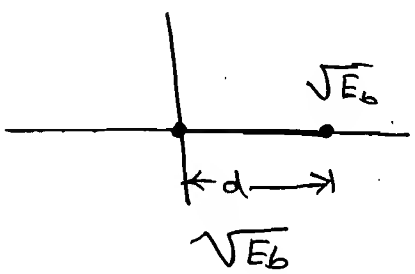
$P_e = Q\left[\sqrt{\frac{2E_b \cos^2\phi}{N_o}}\right]$

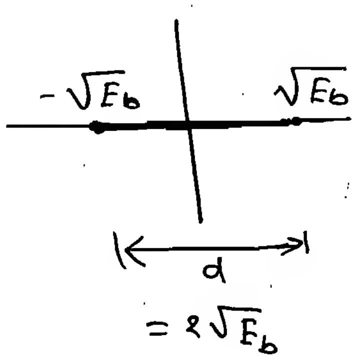
$P_e = Q\left[\sqrt{\frac{E_b \cos^2\phi}{N_o}}\right]$

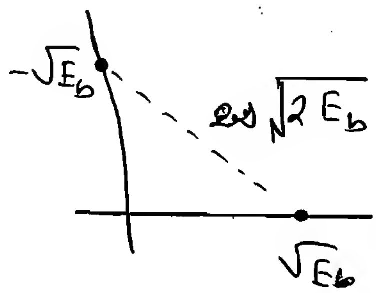
⇒ Due to phase shift in the Local oscillator the Prob. of error increases.

* **ASK** **PSK** **FSK.**

⇒



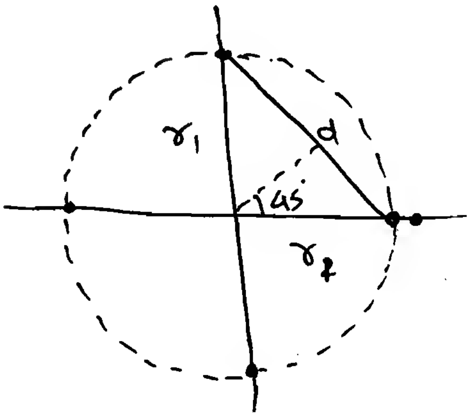




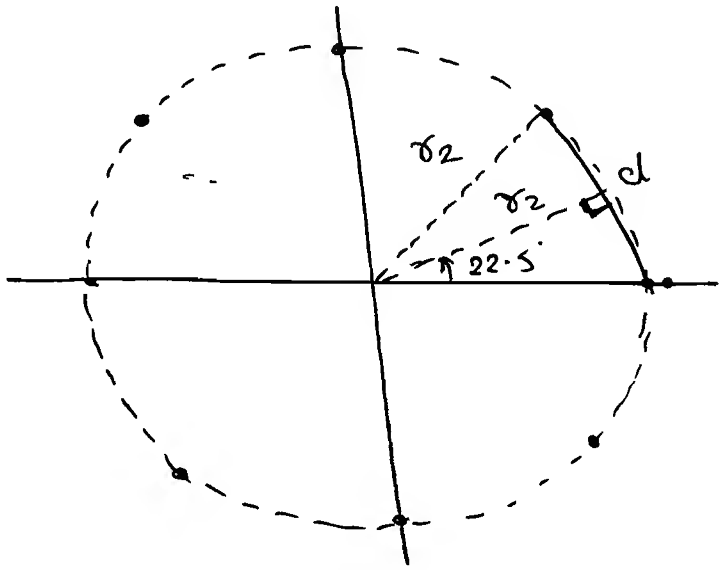
⇒ Based on the Constellation diagram the Prob. of error is.

$$\therefore \boxed{P_e = Q\left[\sqrt{\frac{d_{min}}{2N_0}}\right]} \leftarrow \text{H.B.}$$

$\boxed{Q}$ Consider the Constellation diagram of QPSK and 8-PSK as shown in figure.



QPSK



8-PSK

$\therefore$ ~~sin 22.5/2~~

$$\sin 45^\circ = \frac{\frac{d}{2}}{r_1}$$

$$\therefore r_1 = \frac{d}{2\sin 45^\circ}$$

$$\therefore E_1 = r_1^2 = \frac{d^2}{4 \times \frac{1}{2}} = \frac{d^2}{2}$$

$$\therefore \sin 22.5^\circ = \frac{\frac{d}{2}}{r_2}$$

$$\therefore r_2 = \frac{d}{2\sin 22.5^\circ}$$

$$\therefore E_2 = r_2^2 = \frac{d^2}{4 \times 0.3827}$$

$$\therefore \frac{E_2}{E_1} = \left(\frac{r_2}{r_1}\right)^2$$

$$\therefore \frac{E_2}{E_1} = \left(\frac{1.307}{0.707}\right)^2$$

$$\therefore 10 \log_{10} \frac{E_2}{E_1} = 10 \log \left(\frac{1.307}{0.707} \right)^2 \text{ dB}$$

$$= 10 \log (1.8486)^2 \text{ dB}$$

$$(E_2)_{dB} - (E_1)_{dB} = 5.33.$$

$$\therefore \boxed{(E_2)_{dB} = 5.33 + (E_1)_{dB}.}$$

☆ Information Theory

* Information:

⇒ Information associated with any event is inversly proportional to probability of occurance.

$$\Rightarrow \quad I \propto \frac{1}{P}$$

$$\Rightarrow \quad \boxed{I = \log_2 \frac{1}{P} \text{ bits}} \Leftarrow \text{H.B.}$$

* Entropy:

⇒ The average amount of information is called Entropy.

$$\boxed{H = \sum_i P_i \log_2 \frac{1}{P_i} \text{ bits/symbol}} \Leftarrow \text{H.B.}$$

⇒ Information and Entropy can be understand by following example.

$\boxed{Q}$ A discrete source generates 4 symbols x_1, x_2, x_3, x_4 with probabilities $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$, & $\frac{1}{8}$ respectively. Calculate the entropy. Calculate the entropy when the symbols occurs with equal probability.

Solⁿ:

$$\Rightarrow \quad H = \sum_i P_i \log_2 \frac{1}{P_i}$$

$$\therefore \quad H = P_1 \log_2 \frac{1}{P_1} + P_2 \log_2 \frac{1}{P_2} + P_3 \log_2 \frac{1}{P_3} + P_4 \log_2 \frac{1}{P_4}.$$

$$\therefore \quad \cancel{H = P_4 \log_2 \frac{1}{2}}$$

$$\Rightarrow \quad H = \frac{1}{2}\log_2 2 + \frac{1}{4}\log_2 4 + \frac{1}{8}\log_2 8 + \frac{1}{8}\log_2 8.$$

$$H = \frac{1}{2}(1) + \frac{2}{4} + 3/8 + 3/8.$$

$$\Rightarrow \quad \boxed{H = 1.75 \text{ bits}|\text{symbol}}$$

$\Rightarrow$ If x_1, x_2, x_3, x_4 have equal probability, then

$$H = 4\left(\frac{1}{4}\log_2 4\right).$$

$$\therefore \quad \boxed{H = 2 \text{ bits}|\text{symbol}.}$$

$\Rightarrow$ Information Theory Concept is used to transmit non-electric discrete signals through a digital Communication.

$\Rightarrow$ There are two types of Coding are used. It is called <u>Source Coding</u>.

$\Rightarrow$ <u>Source</u> <u>Coding</u> $\rightarrow$ <u>Fixed length</u> Coding.

$\qquad\qquad\qquad\quad \rightarrow$ <u>Variable length</u> Coding.

⇒ In Early days morse code is used to transmit data. i.e. • & — are transmitted as a data.

① Fixed length Coding:

⇒

26	A	00000
	B	00001
	C	⋮
	⋮	⋮
	X	⋮
	Z	⋮

26 alphabets

so, $2^n = 26$

⇒ $n = 5$ bits required for each symbol.

⇒ In fixed length Coding, Code length is fixed for every symbol. i.e. code is independent of the probability of occurance of that symbol.

② Variable Length Coding:

⇒ In Variable Length Coding, Length of Code is not fixed. It is depend on the probability of occurance of that Code.

Let, 4 symbols x_1, x_2, x_3, x_4 having prob. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}$ respectively.

x_1	$\frac{1}{2}$	0
x_2	$\frac{1}{4}$	10
x_3	$\frac{1}{8}$	110
x_4	$\frac{1}{8}$	111

⇒ First arrange the prob. in assending order.

⇒ Now, draw a line in a such a way that the sum of prob. of symbols uper to the line is exactly equal to the sum of prob. of symbols below the line.

⇒ Now, give code '0' to the upper symbols and '1' to the Lower symbols.

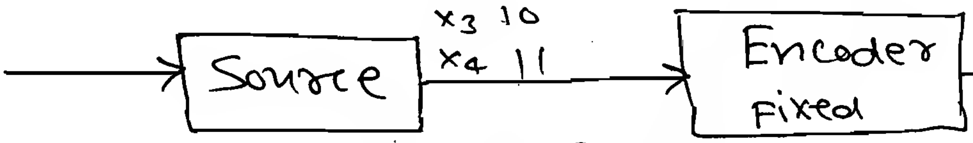
⇒ Now, the symbol which is have unique symbol leave it and Repeate process for the remaing symbol untill unless we get the unique code for each and every symbol.

⇒ Entropy indicates the average no. of bits required to encode the each symbol if variable Length Coding is used.

⇒

x_1	00
x_2	01
x_3	10
x_4	11

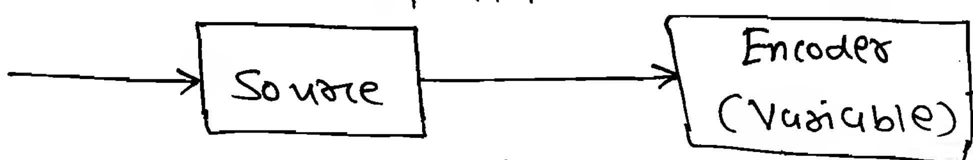
$(L = 2)$ 2 bits/symbol



$\boxed{2000 \text{ bits}}$

Let, 1000 Symbols

		symb.		
x_1	0	500	⇒	500 × 1 = 500 bits
x_2	10	250	⇒	250 × 2 = 500 bits
x_3	110	125	⇒	125 × 3 = 375 bits.
x_4	111	125	⇒	125 × 3 = 375 bits



$\boxed{1750 \text{ bits}}$

* Information rate & Bit rate.

$$\Rightarrow \text{Bit rate} = \frac{\text{bits}}{\text{sec}} = \underbrace{\frac{\text{bits}}{\text{symbol}}}_{\text{Entropy}} \times \underbrace{\frac{\text{Symbol}}{\text{sec}}}_{\text{symbol rate}}.$$

$$\Rightarrow R_b = [E \times \text{Symbol rate}] \quad \text{bits/sec.}$$

$$\therefore \boxed{R_b = \text{Entropy} \times \text{symbol rate.} \quad \text{bits/sec.}}$$

H

* Efficiency of code:

$$\boxed{\eta = \frac{H}{\bar{L}}, \quad \bar{L} = \sum P_i L_i}$$

$\Rightarrow$ for previous example,

$$H = 1.75 \text{ bits/symbol}$$

$$\bar{L} = \sum_i P_i L_i = \left(\tfrac{1}{2} \times 1\right) + \left(\tfrac{1}{4} \times 2\right) + \left(\tfrac{1}{8} \times 3\right) + \left(\tfrac{1}{8} \times 3\right)$$

$$\bar{L} = 1.75 \text{ bits/symbol.}$$

$$\therefore \eta = \frac{1.75}{1.75}$$

$$\Rightarrow \boxed{\eta = 100\,\%}$$

$\Rightarrow$ But in all case, it is not possible that $\eta = 100\,\%$ as it is not possible to divide the symbol above and below line with equal probability.

for e.g. 0.55 & 0.45. so $\boxed{\eta < 100\,\%.}$

Q GATE - 2013

Consider two discrete source X & Y each generating the symbol 1 & -1 with equal probability.

Let, $Z = X + Y$. Determine the entropy of Z.

Solⁿ:

X	Y		Z = X + Y	
1	1		Z	P
-1	-1	(1+1)	2	1/4
		(1-1, -1+1)	0	1/2
		(-1,-1)	-2	1/4.

∴ So, $H = \sum_i P_i \log_2 \frac{1}{P_i}$.

$$\therefore H = \frac{1}{4}\log_2 4 + \frac{1}{2}\log_2 2 + \frac{1}{4}\log_2 4.$$

$$\therefore H = \frac{2}{4} + \frac{1}{2} + \frac{2}{4}.$$

$$\therefore H = 3/2$$

$$\therefore \boxed{H = 1.5 \text{ bits/symbols.}}$$

[a] A discrete Source generate 3 Symbols x_1 x_2 x_3 with Prob. $\frac{1}{2}, \frac{1}{4}, \frac{1}{4}$. The Symbol rate is 3000 symbols/sec. Determine the information rate.

Soln:

$$H = \frac{1}{2}\log_2 2 + \frac{1}{4}\log_2 4 + \frac{1}{4}\log_2 4$$

$$H = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 1.5 \text{ bits/symbols.}$$

$\therefore$ R_b = [Entropy × symbol rate] bits/sec.

$$\therefore R_b = 1.5 \times 3000$$

$$= 4500 \text{ bits/sec}$$

$$\therefore \boxed{R_b = 4.5 \text{ Kbps}}$$

*

$\Rightarrow$ Consider a discrete source generating a two Symbol x_1 & x_2.

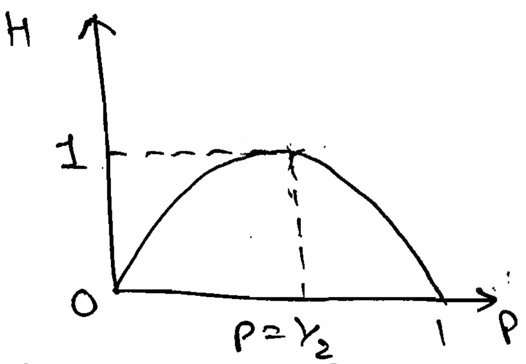
$x_1 \rightarrow P$ $\frac{1}{2}$.

$x_2 \rightarrow (1-P)$ $\frac{1}{2}$.

$$\therefore \quad H = \frac{1}{P}\log_2 P + \frac{1}{1-P}\log_2(1-P).$$

for max H,

$$\frac{dH}{dP} = 0 \Rightarrow \boxed{P = \frac{1}{2}}$$



⇒ The entropy will be max. only when the symbol occurs with equal probability

↗ HB → ∴ $\boxed{H_{max} = \log_2 M.}$

$M = 2 \Rightarrow H_{max} = 1$

M = no. of symbols.

$M = 4 \Rightarrow H_{max} = \log_2 4 = 2.$

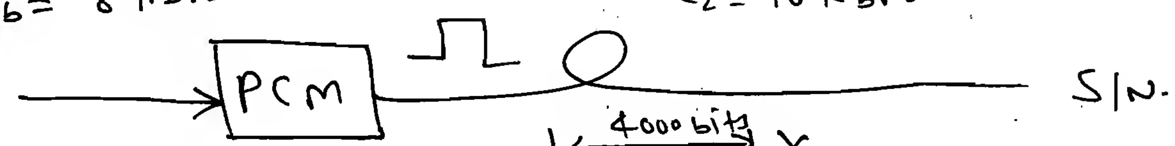
☆ Channel Capacity:

Defn: ⇒ Channel Capacity is defined as the max. no of bits that the channel is capable of transmitting without any error.

⇒ $R_b = 8$ kbps

Let, $C_1 = 4$ k bps ✗

$C_2 = 10$ k bps ✓



8000 bits in 1 sec

4000 bits in 1 sec ✗

10000 bits in 1 sec ✓

∴ $C < R_b$ errors

$C \geq R_b \rightarrow$ No errors.

⇒ To determine the channel capacity Shanon - Hartly Law is used,

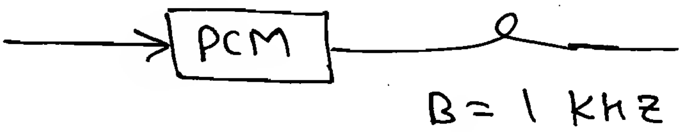
linear scale

i.e. $$\boxed{C = B \log_2 \left[1 + S/N \right] \quad \text{bits/sec.}}$$

⇒ Channel Capacity is directly proportional to the channel BW.

⇒ Channel Capacity is also called data transfer rate.

e.g. :



Let, $S/N = 15$.

① $B = 1$ KHz

⇒ $C = B \log_2 (1 + S/N)$.

$C = 10^3 \log_2 (1 + 15)$.

∴ $\boxed{C = 4 \text{ Kbps.}}$

Twisted Pair

② $B = 1$ MHz.

⇒ $C = 10^6 \log_2 (1 + 15)$.

$\boxed{C = 4 \text{ Mbps}}$

Coaxial Cable

③ $B = 1$ GHz.

⇒ $\boxed{C = 4 \text{ Gbps}}$ → FOC.

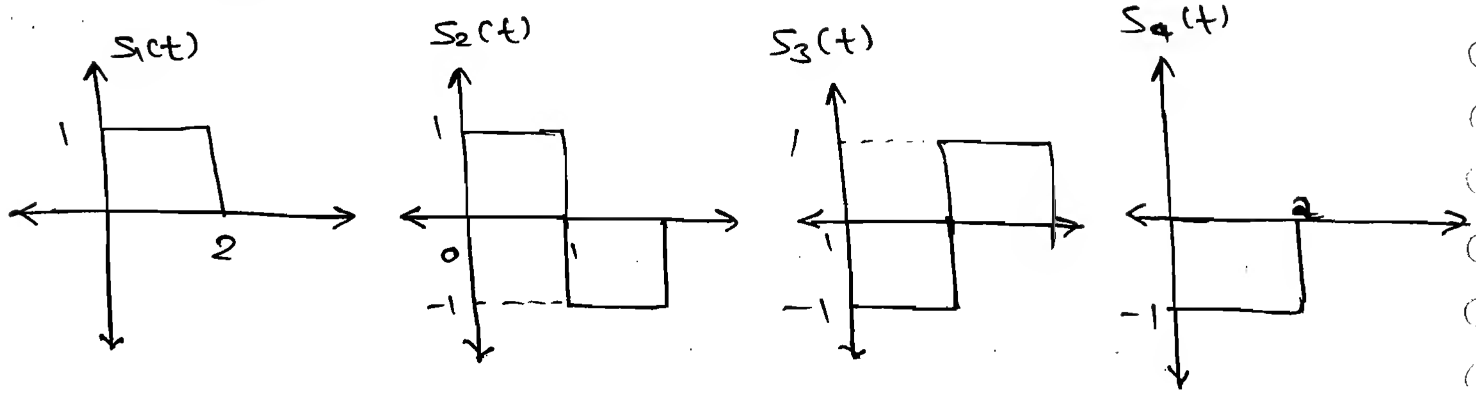
⇒ When $B \to \infty$ ⇒ $$\boxed{C_{max} = 1.44 \frac{S}{N_o}}$$

signal power

PSD of Noise

* Constellation diagram Concept:

⇒ Consider the four signal as shown in figure.



⇒ The above four signals can be represented as a linear combination of orthonormal basis f^n.

⇒ The orthonormal basis function should satisfy the following two properties:

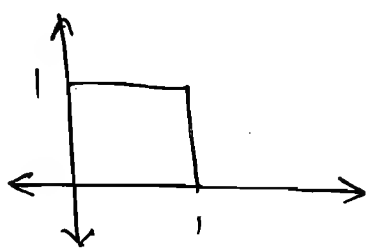
① The energy should be 1.

$$\int_{-\infty}^{\infty} \phi_1(t)\,dt = 1, \quad \int_{-\infty}^{\infty} \phi_2(t) = 1.$$

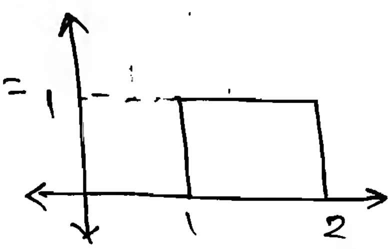
② They should be orthogonal to each other.

⇒ So, for this case $\phi_1(t)$ & $\phi_2(t)$ are as follow:

⇒ $\phi_1(t) =$



, $\phi_2(t) =$



$\Rightarrow$ So,

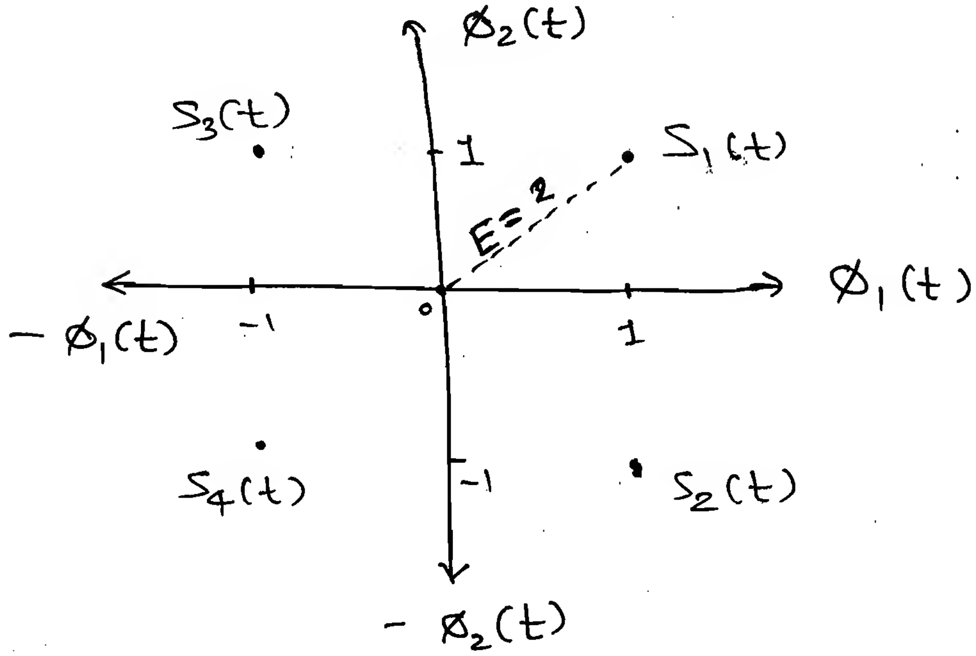
$$S_1(t) = 1 \cdot \phi_1(t) + 1 \cdot \phi_2(t).$$

$$S_2(t) = 1 \cdot \phi_1(t) - 1 \cdot \phi_2(t).$$

$$S_3(t) = -1 \cdot \phi_1(t) + 1 \cdot \phi_2(t).$$

$$S_4(t) = -1 \cdot \phi_1(t) - 1 \cdot \phi_2(t).$$

$\Rightarrow$



(signal space diagram (or) Constellation diagram).

$\Rightarrow$ In, general $E = 1$.

so, the signal s(t) should be

$$\boxed{\sqrt{\frac{2}{T_b}} \cdot \cos 2\pi f_c t \Rightarrow E = 1} \Leftarrow \text{H.B.}$$

as, $E = \frac{A^2 T_b}{2} = 1$

$$A = \sqrt{\frac{2}{T_b}} \cdot \quad \Rightarrow \quad A_c \cos 2\pi f_c t$$

$$= \sqrt{\frac{2}{T_b}} \cdot \cos 2\pi f_c t$$

$$\Rightarrow E = 1.$$

So, $\phi_1(t) = 1 \cdot \sqrt{\frac{2}{T_b}} \cdot \cos 2\pi f_c t \Rightarrow E = 1.$

& $\phi_2(t) = 1 \cdot \sqrt{\frac{2}{T_b}} \cdot \sin 2\pi f_c t \Rightarrow E = 1.$

} Standard form.

⇒ <u>ASK:</u>

$$S_1(t) = A_c \cos 2\pi f_c t \quad '1'$$
$$= 0 \quad '0'.$$

$$E = \int_0^{T_b} |s_1(t)|^2 \cdot dt \Rightarrow E_b = P \cdot T_b$$

$$E_b = \frac{A_c^2}{2} \cdot T_b.$$

$$A_c = \sqrt{\frac{2E_b}{T_b}}.$$

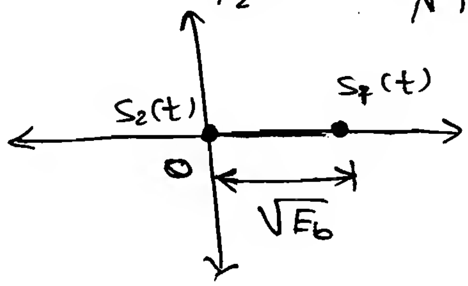
$$\therefore S_1(t) = \sqrt{\frac{2E_b}{T_b}} \cdot \cos 2\pi f_c t \quad , '1'$$

$$S_2(t) = 0 \quad , '0'.$$

$$\Rightarrow S_1(t) = \sqrt{E_b} \cdot \sqrt{\frac{2}{T_b}} \cdot \cos 2\pi f_c t.$$

$$S_2(t) = 0.$$

⇒ $\phi_2(t) = \sqrt{\frac{2}{T_b}} \cdot \sin 2\pi f_c t$



$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cdot \cos 2\pi f_c t$

⇒ $\phi_1(t)$ & $\phi_2(t)$ in a such way that $E = 1.$

So, $$\boxed{\begin{aligned} \phi_1(t) &= \sqrt{\frac{2}{T_b}} \cdot \cos 2\pi f_c t \\ \phi_2(t) &= \sqrt{\frac{2}{T_b}} \cdot \sin 2\pi f_c t. \end{aligned}}$$

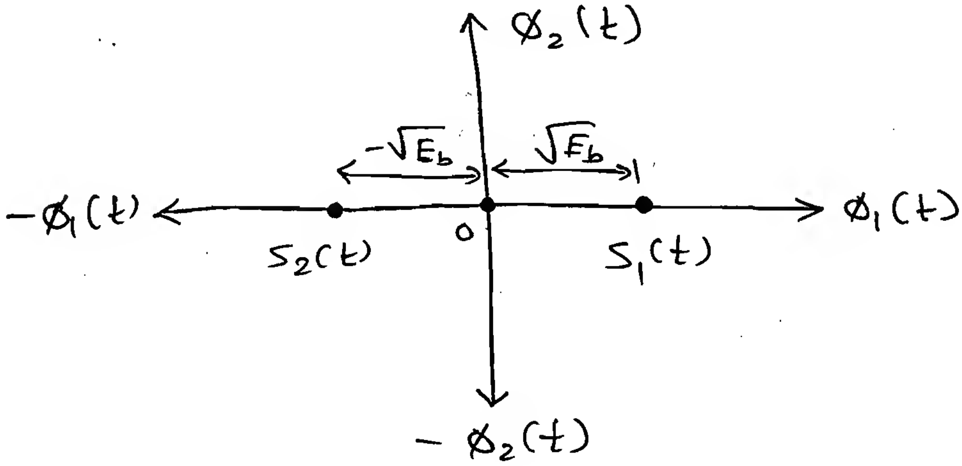
⇒ **For BPSK:**

$$S_1(t) = +\sqrt{E_b}\cdot\sqrt{\frac{2}{T_b}}\cdot\cos 2\pi f_c t \qquad \text{'1'.}$$

$$S_2(t) = -\sqrt{E_b}\cdot\sqrt{\frac{2}{T_b}}\cdot\cos 2\pi f_c t \qquad \text{'0'.}$$

$$\Rightarrow \quad S_1(t) = +\sqrt{E_b}\cdot\phi_1(t) + 0\cdot\phi_2(t)$$

$$S_2(t) = -\sqrt{E_b}\cdot\phi_1(t) + 0\cdot\phi_2(t).$$



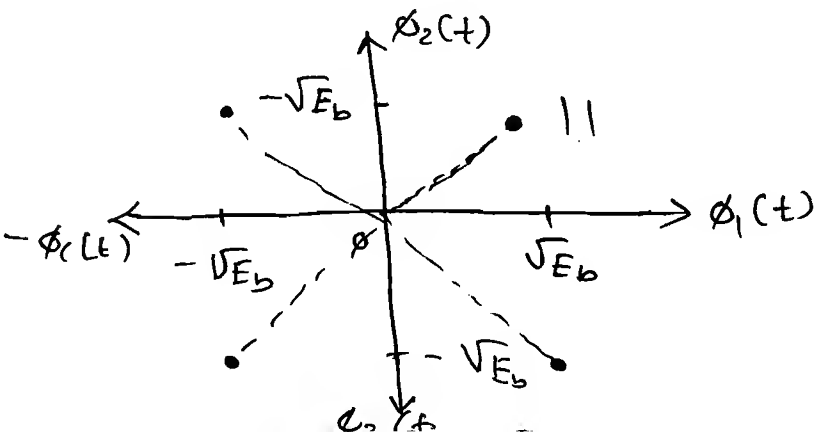
⇒ **For QPSK:**

$$S_1(t) = +\sqrt{E_b}\cdot\phi_1(t) + \sqrt{E_b}\,\phi_2(t) \Rightarrow 11$$

$$S_2(t) = +\sqrt{E_b}\cdot\phi_1(t) - \sqrt{E_b}\,\phi_2(t) \Rightarrow 10$$

$$S_3(t) = -\sqrt{E_b}\cdot\phi_1(t) + \sqrt{E_b}\,\phi_2(t) \Rightarrow 01$$

$$S_4(t) = -\sqrt{E_b}\,\phi_1(t) - \sqrt{E_b}\,\phi_2(t) \Rightarrow 00$$



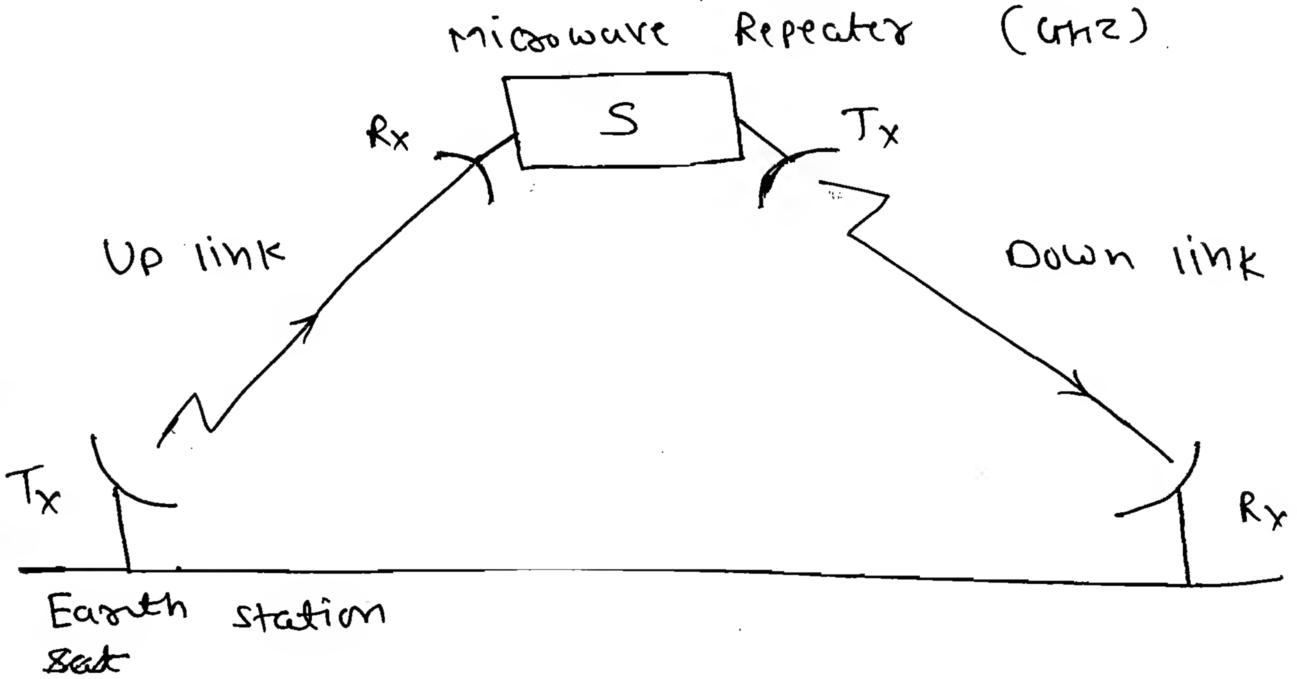
Multiple Access Techniques:

⇒ (1) FDMA
(2) TDMA
(3) CDMA
} Wireless Comm.

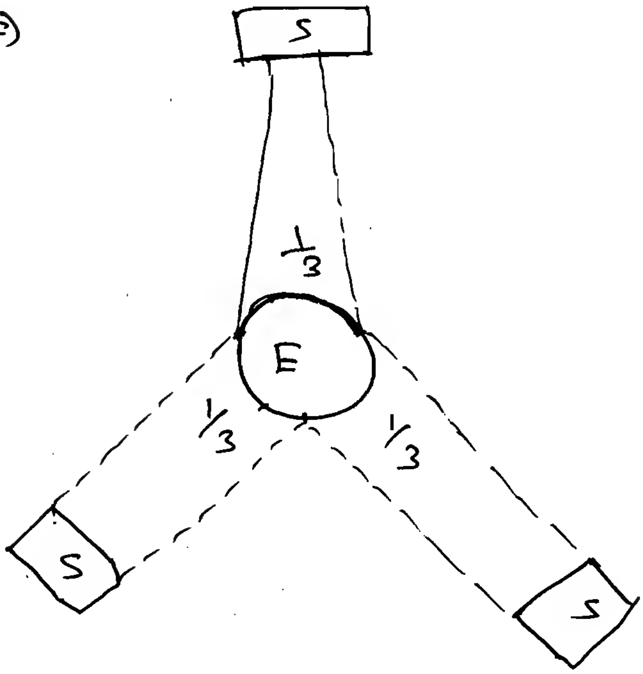
FDM
TDM
} Wired Comm.

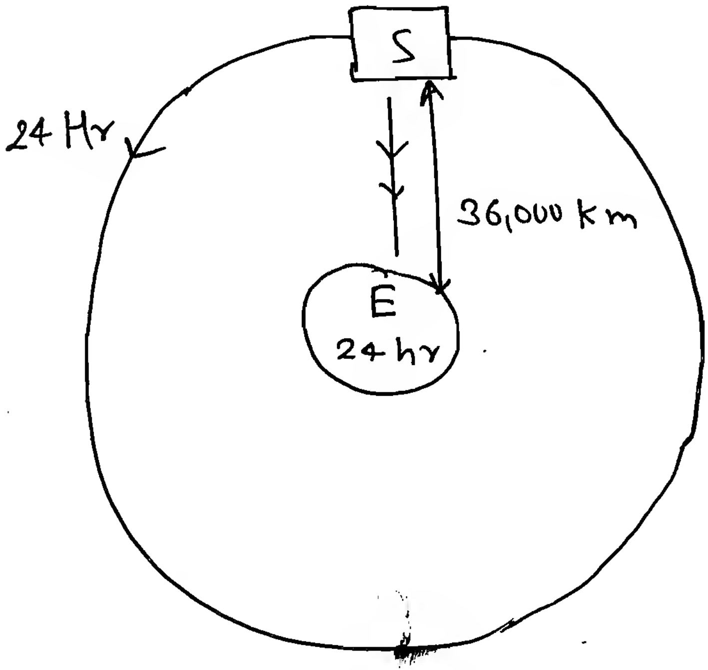
⇒ Consider the example of Satellite Communication to understand the above techniques.

⇒

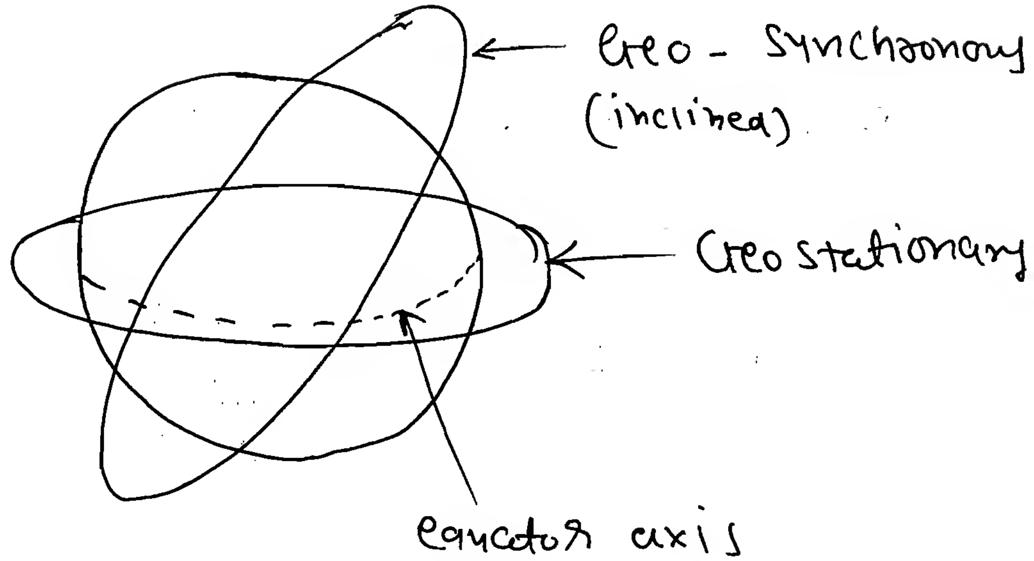


⇒





⇒ Geo - stationary and geo - synchronous:



⇒ Through out the World, 3 combinations for Uplink & Down Link are used:

6/4 GHz.

14/12 GHz.

30/20 GHz.

Q Why Down link is not same as Uplink?

⇒ If Down link is same as the Uplink. then interference are occures. If

UL = DL = 6 GHz.

⇒ Then, when R Tx of satellite transmit the data it may be received by its own Receiver. So, interference will be occured.

$\boxed{Q}$ Why Uplink should be greater than the Downlink?

Ans: Power received by the Rx Es is given by,

$$P_r = 10\log\left[\frac{P_t \cdot G_t \cdot G_r}{\left(\frac{4\pi R}{\lambda}\right)^2}\right].$$

$$\Rightarrow P_r = \underbrace{10\log[P_t G_t]}_{Tx} + \underbrace{10\log[G_r]}_{Rx} - \underbrace{10\log\left[\left(\frac{4\pi R}{\lambda}\right)^2\right]}_{\text{Path loss}}.$$

$\Rightarrow$ As the Satellite produces power from the solar array, Power produce is limitted. one way to increase received power is by reducing the path loss. In order to reduce the path loss, λ should be increase and as $\lambda = \frac{c}{f}$, freq. should be decrease. That's why down link should be less than the up link.

* **Disadvantages:**

① very large loss as $R = 36,000$ km.

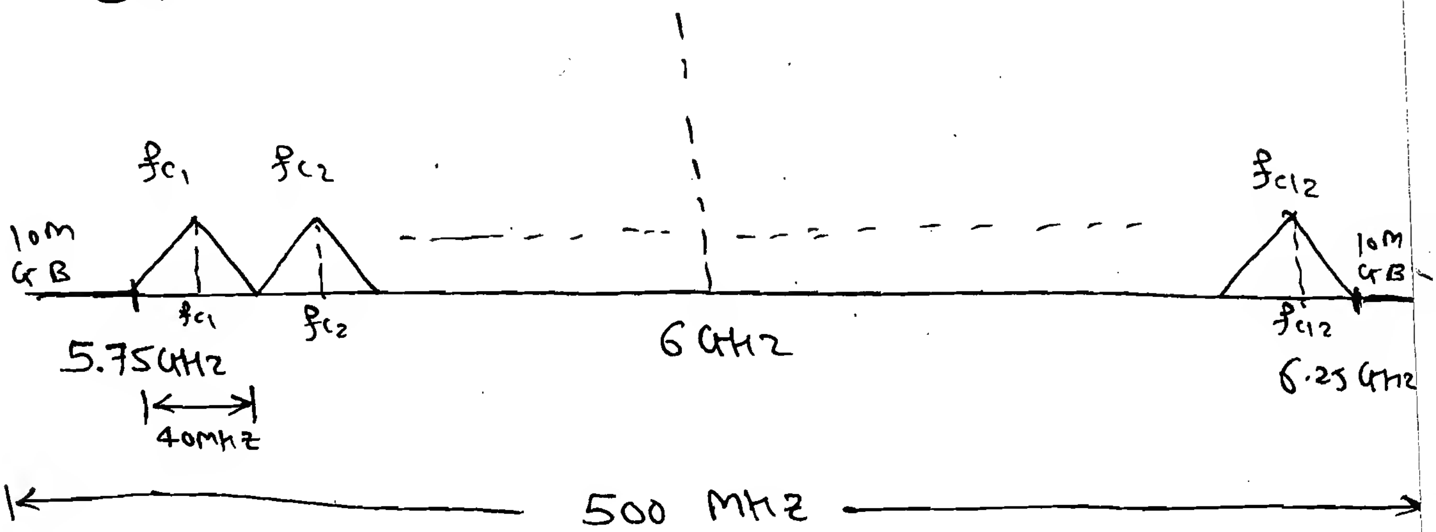
② Delay bet^n Tx & Rx signal as $R = 36,000$ km

$$T = \frac{R}{C} = \frac{36,000 \times 10^3}{3 \times 10^8} = 0.12 \text{ sec.}$$

① FDMA: (Frequency Division Multiple Access).

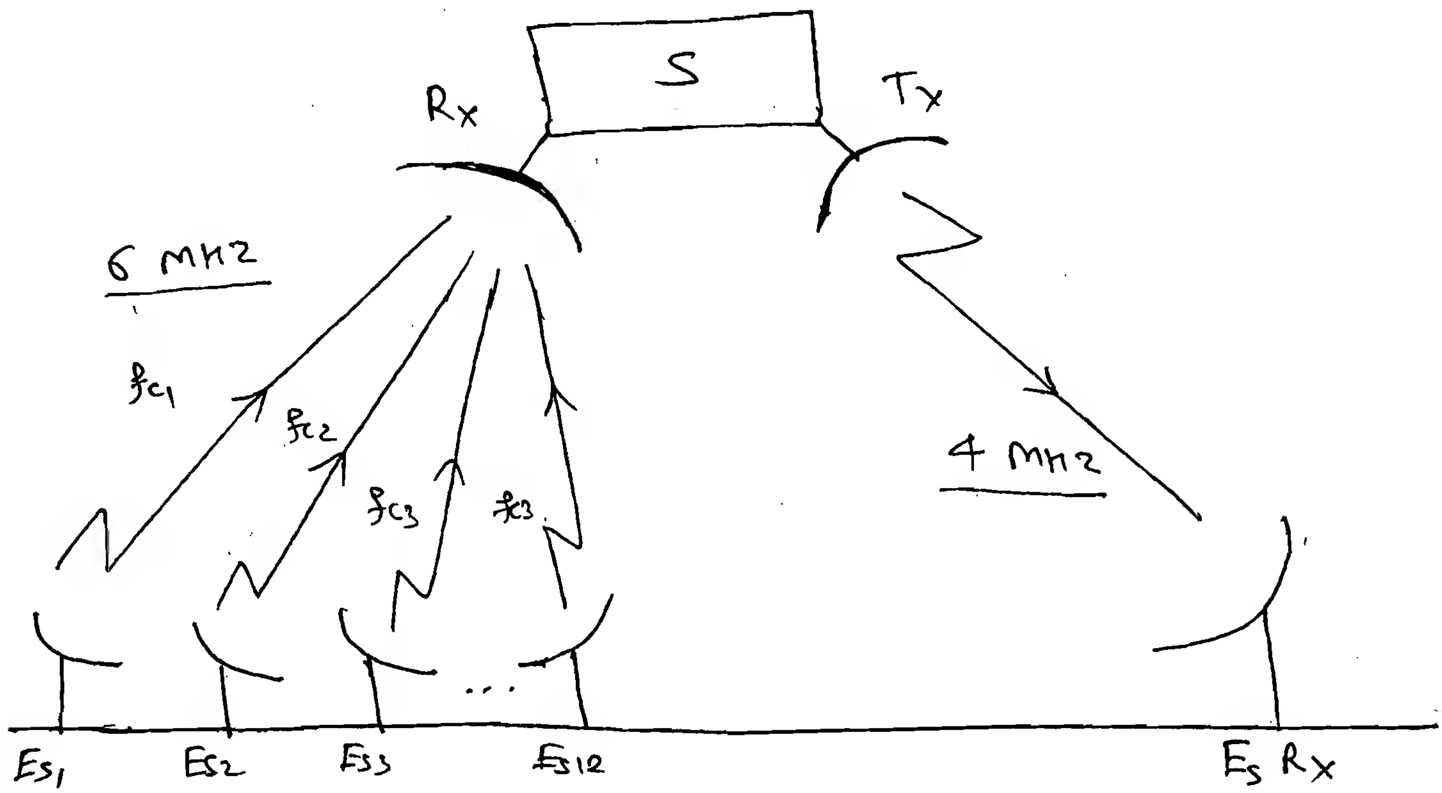
⇒ In FDMA, Multiple users will access the Satellite with the allotted carrier freq.

⇒ BW = 500 MHz.

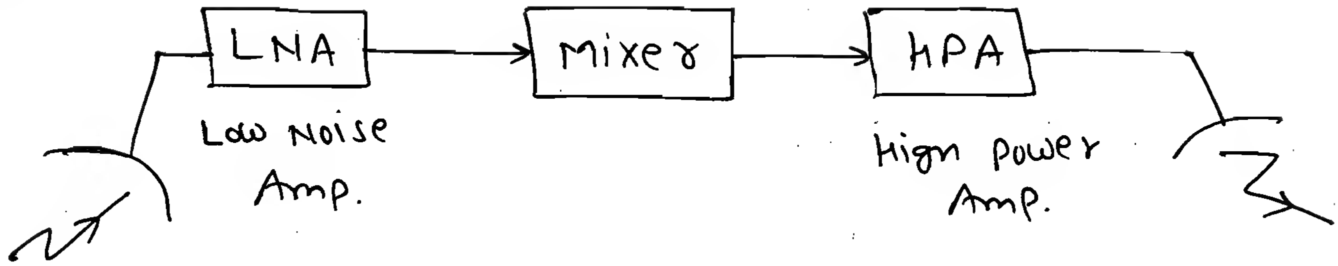


⇒ 1st satellite INTELSAT-1 by USA.

⇒ India 1st Satellite INSAT-1

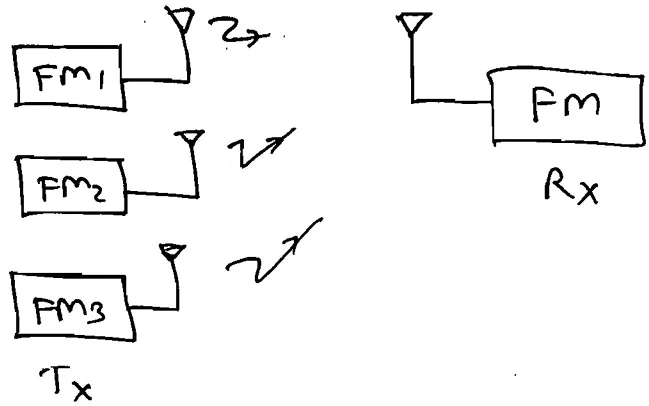


⇒



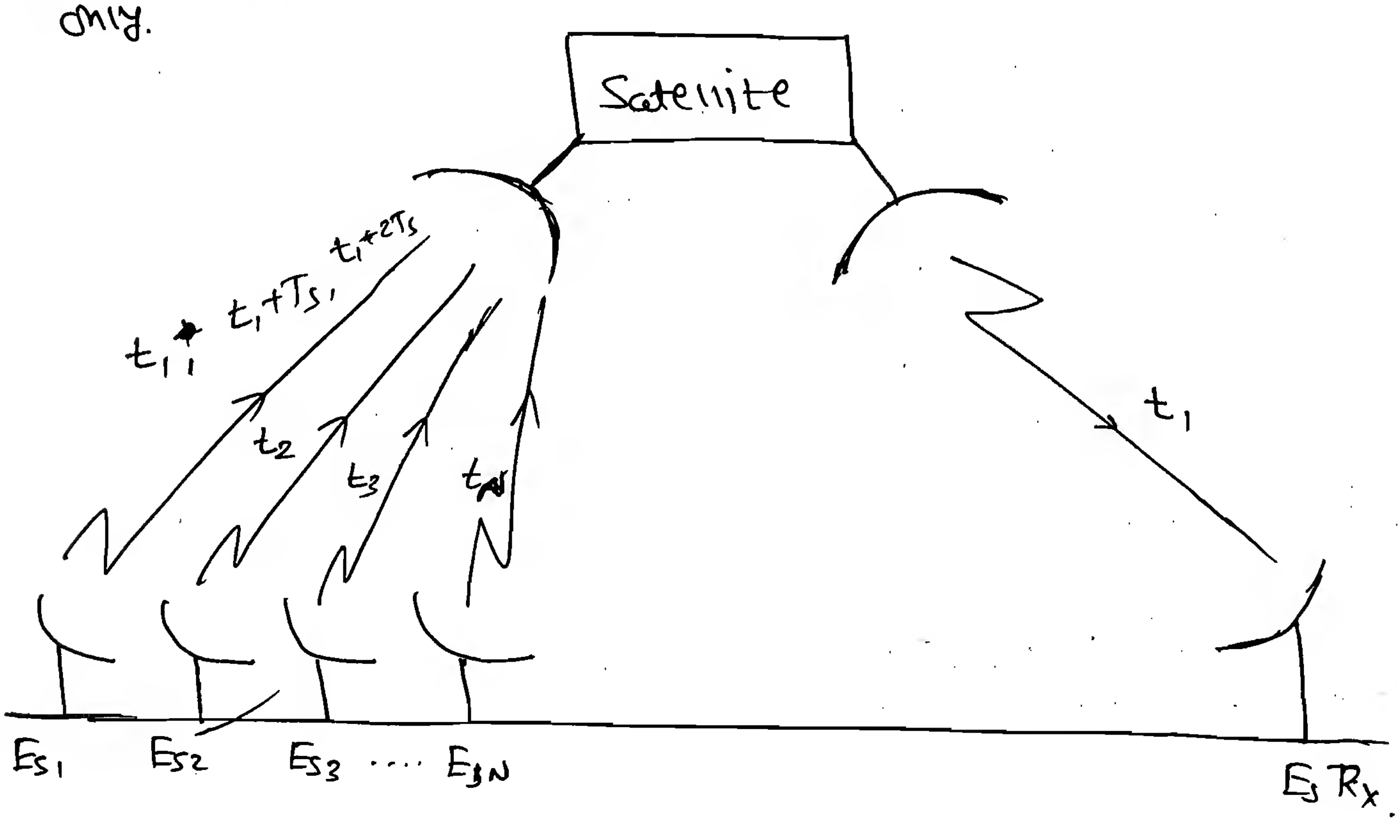
Transponder.

⇒ Another example is: Radio

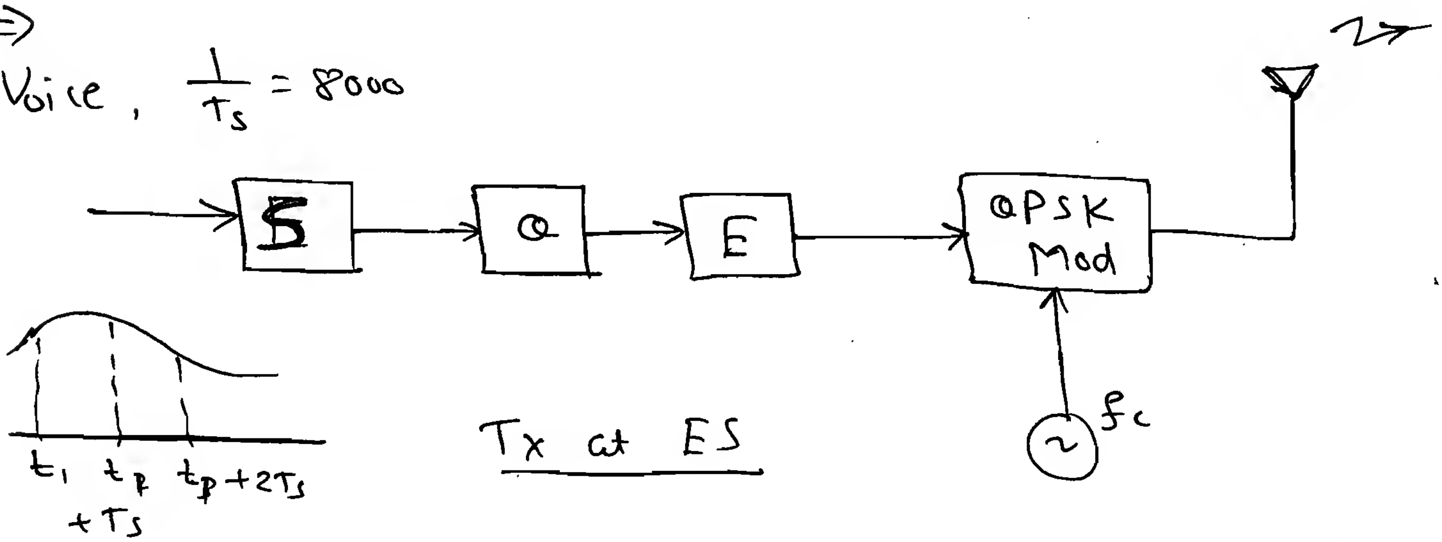


② TDMA : (Time Division Multiple Access).

⇒ In TDMA multiple users will access the satellite in the allotted time slot only.



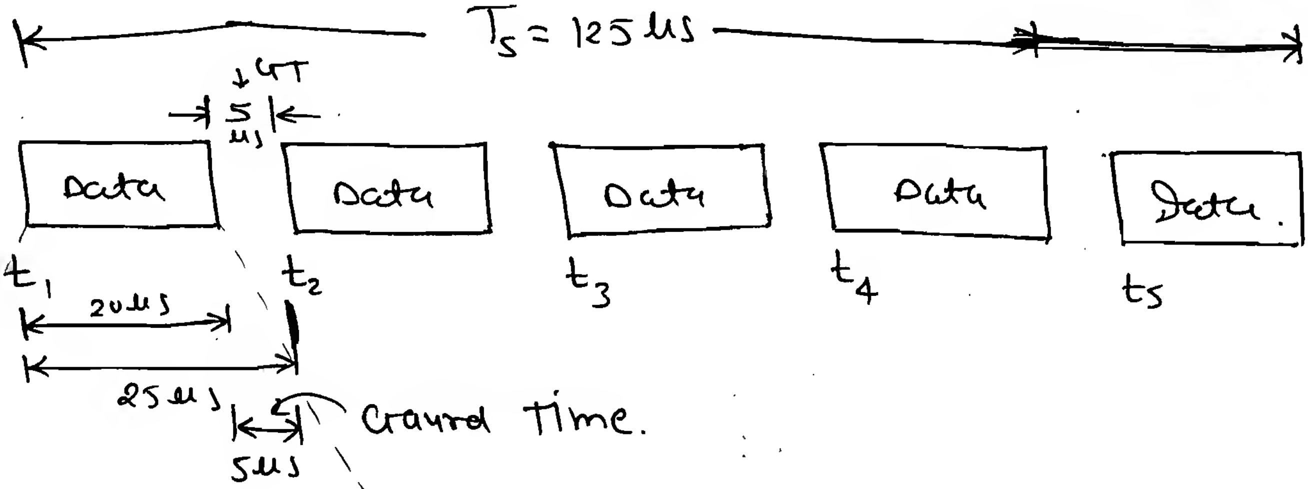
⇒ Voice, $\frac{1}{T_s} = 8000$

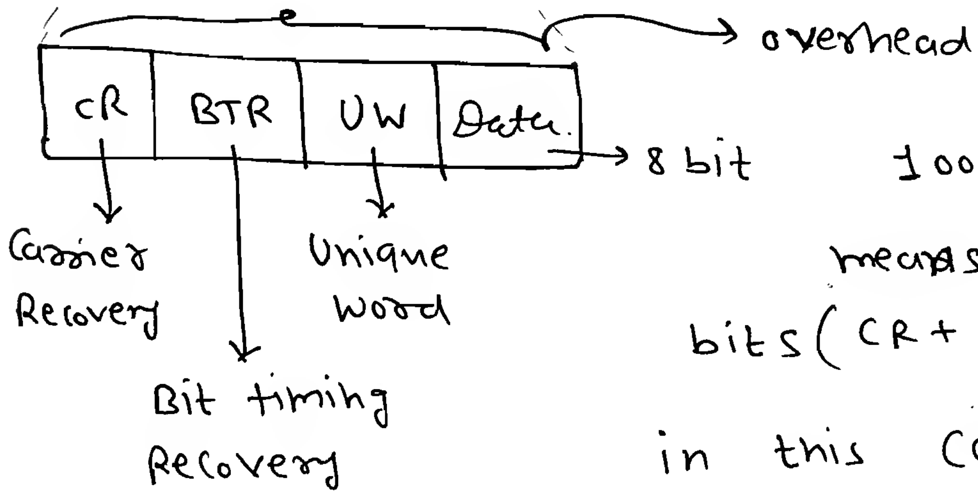


Tx at ES

$$\Rightarrow \quad \frac{1}{T_s} = 8000 \Rightarrow T_s = 125\,\mu s.$$

⇒ TDMA Frame:





100 % overhead bits means

bits(CR + BTR + UW) = (Data bits).

in this case

Data bits = 8

So, 100 % over head bits = 8.

and Total bit in 1 Frame = 8 + 8 = 16.

Procedure:

(1) $T_s = ?$

$T_s = 125\ \mu s.$

(2) No. of ES = 5.

(3) Time slot to each earth station

$$T_1 = \frac{125\ \mu s}{5} = 25\ \mu sec = \frac{(1)}{(2)}.$$

(4) $T_1 - GT = 25 - 5 = 20\ \mu sec.$

as $GT = 5\ \mu sec.$

(5) No. of over head bits + Data bits

$= 12 + 8 = 20$ bits.

(6) $$T_b = \frac{20\ \mu sec}{20\ bits} = 1\ \mu sec/bits = \frac{(4)}{(5)}$$

(7) $$R_b = \frac{1}{T_b}$$

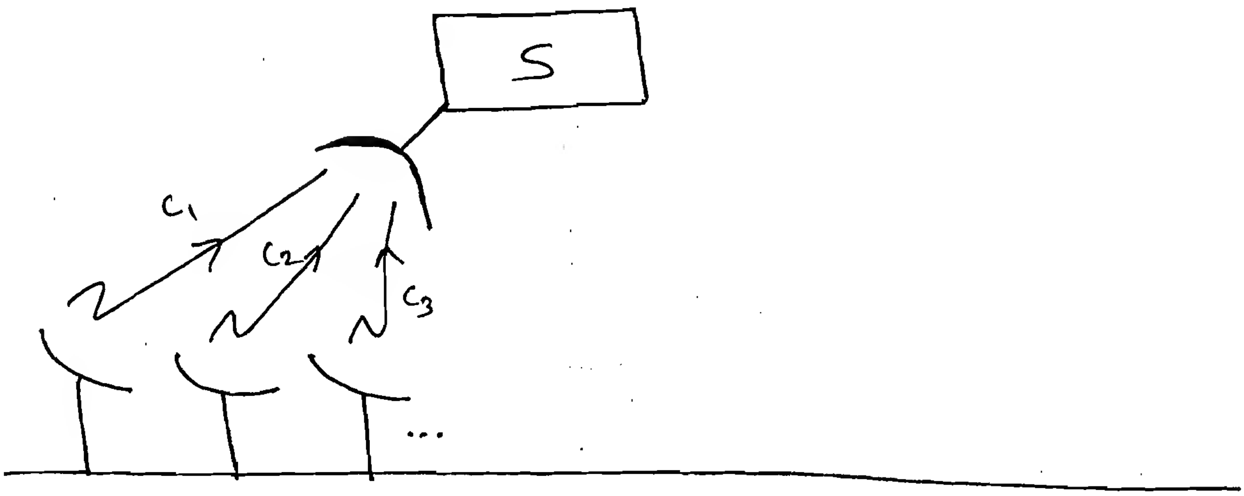
$$\Rightarrow R_b = \frac{1}{1\ \mu sec/bits}$$

$$\Rightarrow \boxed{R_b = 1\ \text{MBPS}}$$

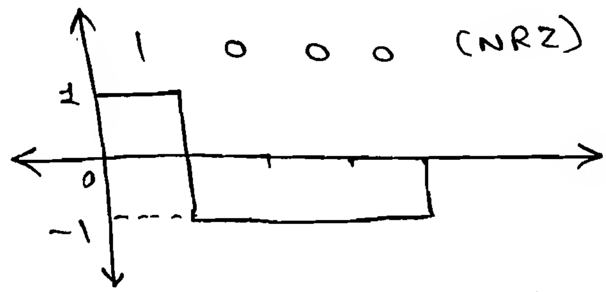
③ CDMA : (Code Division Multiple Access):

⇒ In CDMA, All the users can access the satellite at the same time and all signal will occupy the same freq. range.

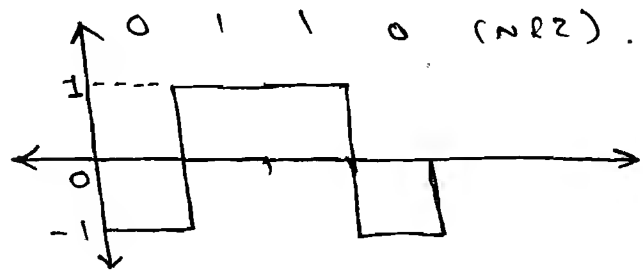
⇒



→ $C_1 = 1\ 0\ 0\ 0$

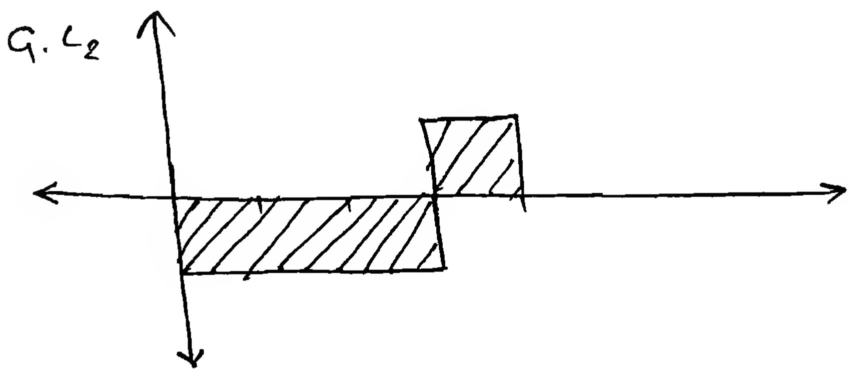


→ $C_2 = 0\ 1\ 1\ 0$



⇒ $\int_0^{4T_b} f_1(t)\cdot f_2(t)\, dt \neq 0.$

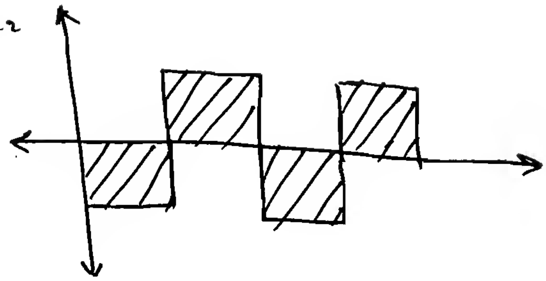
So, not orthogonal to each other.



⇒ if $C_1 = 1\,0\,0\,0$

$C_2 = 0\,0\,1\,0$

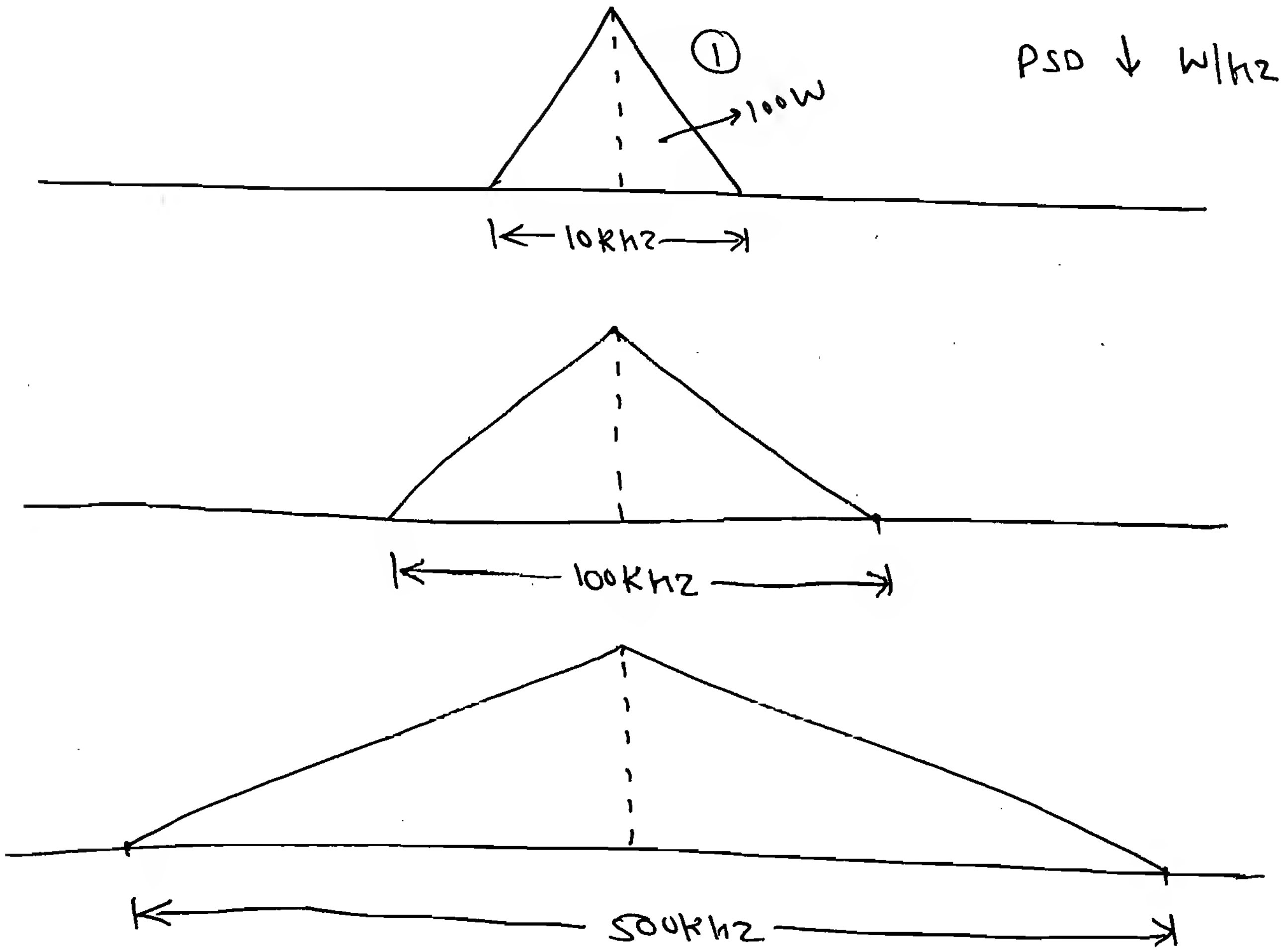
then $\int_0^{4T_b} f_1(t) \cdot f_2(t)\, dt = 0.$



* Spread Spectrum modulation:

⇒

PSD ↓ W/Hz



⇒ **PN Sequence Code:**

Property: ① All codes are orthogonal to each other.

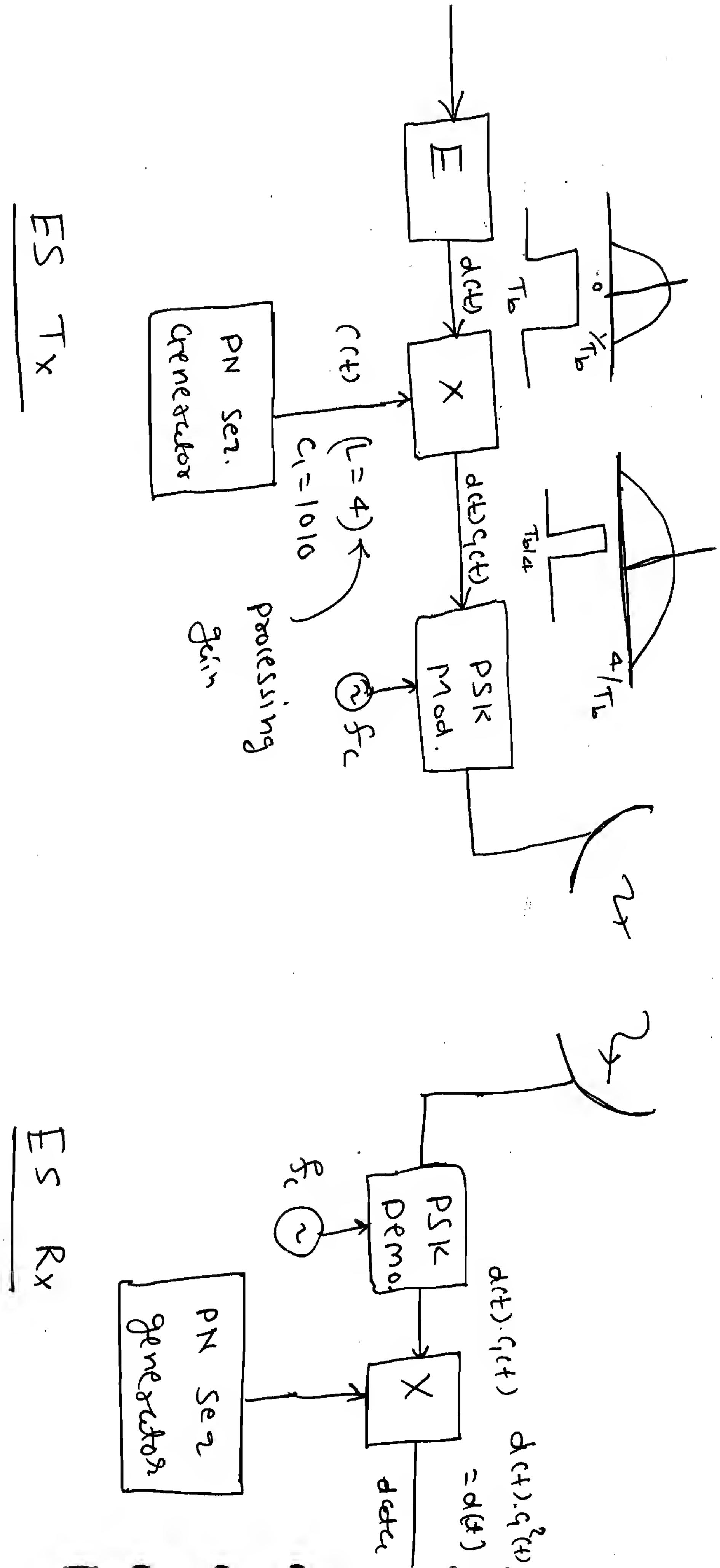
② $C_1^2(t) = 1$

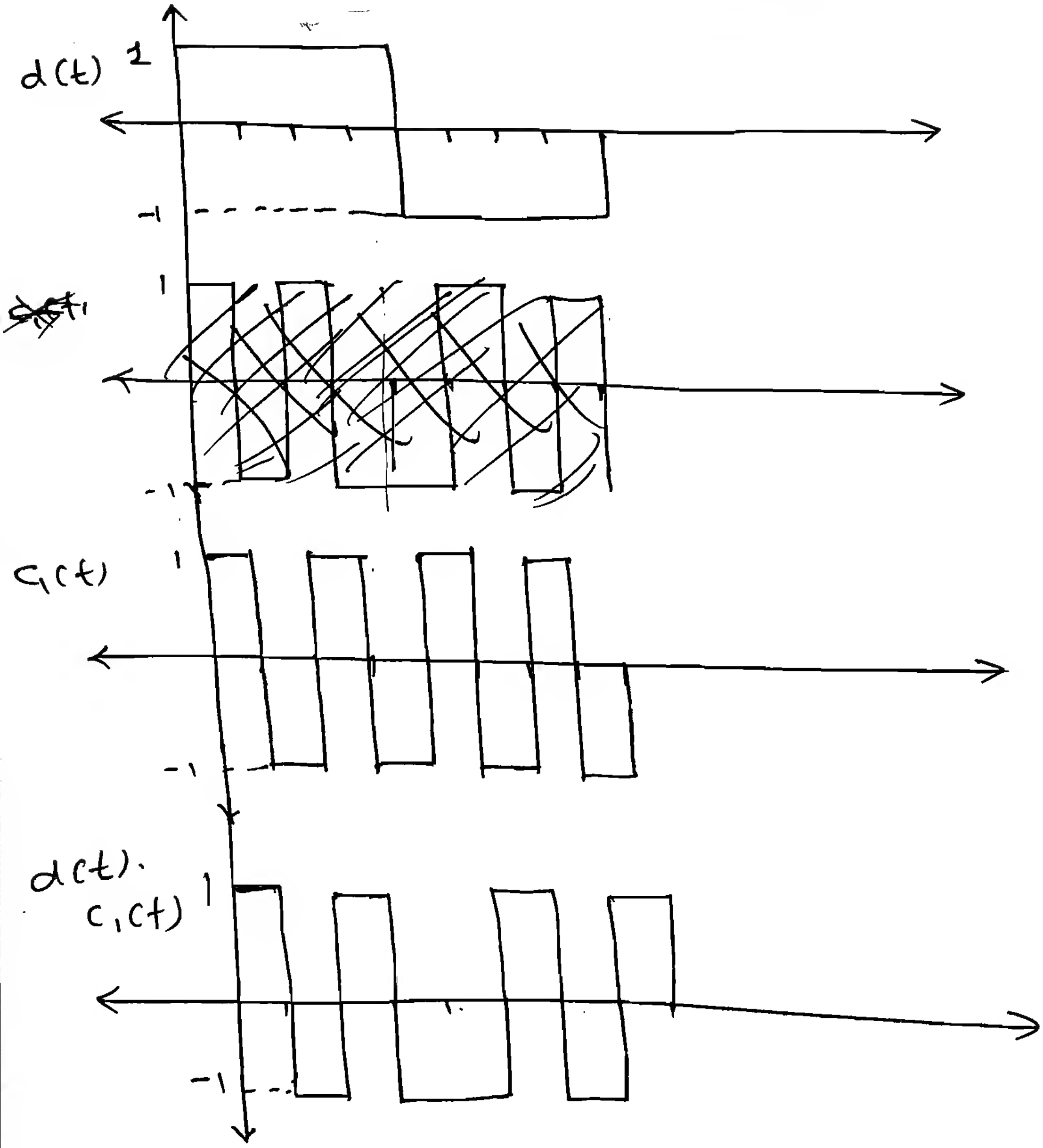
$C_2^2(t) = 1$

$C_3^2(t) = 1$

⋮

* CDMA Block Diagram:





☆ **GSM System:-** (Global system for Mobile communication).

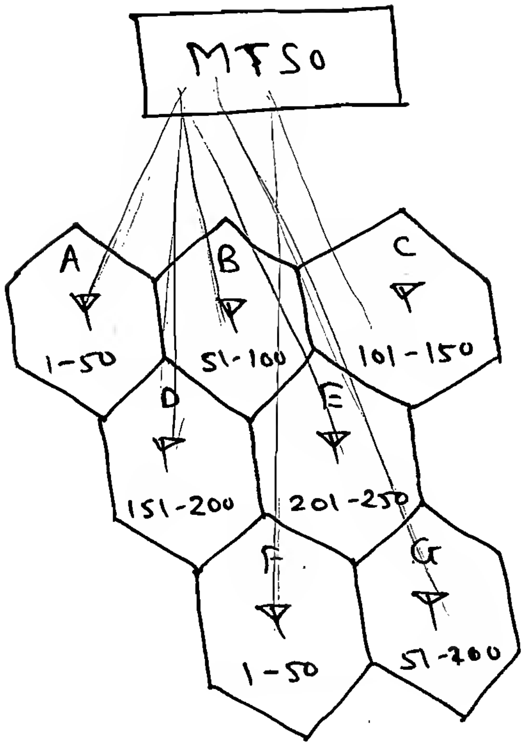
⇒ 800 MHZ

⇒ Global system for Mobile Communication.

⇒ Area Coverage is limited but frequency Reuse is possible.

⇒ Working Principle of GSM System:

⇒



A & F = Co-Channel cell site

⇒ Let, Considered allotted spectrum to the cellular operator is 1 MHz.

$$BW = 1 \text{ MHz}.$$

$$\text{Voice channel } BW = 4 \text{ kHz}.$$

$$\Rightarrow \text{Voice channel} = \frac{1 \text{ MHz}}{4 \text{ kHz}} = 250.$$

⇒

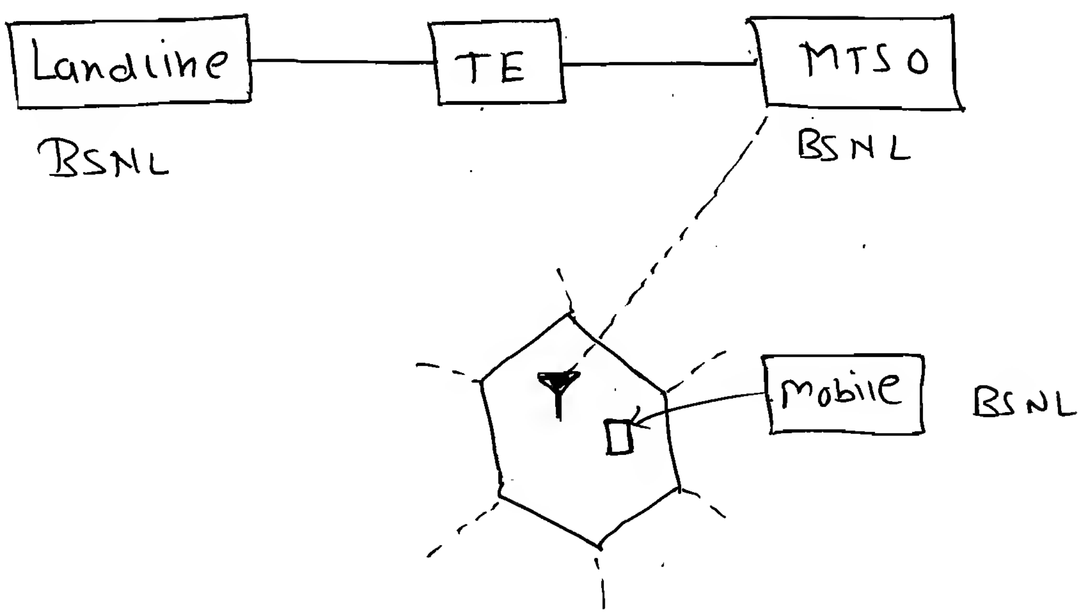
⇒ Let, freq. reuse factor is 5 i.e $K = 5$.

Voice channel		Group
1 – 50	—	1
51 – 100	—	2
101 – 150	—	3
151 – 200	—	4
201 – 250	—	5

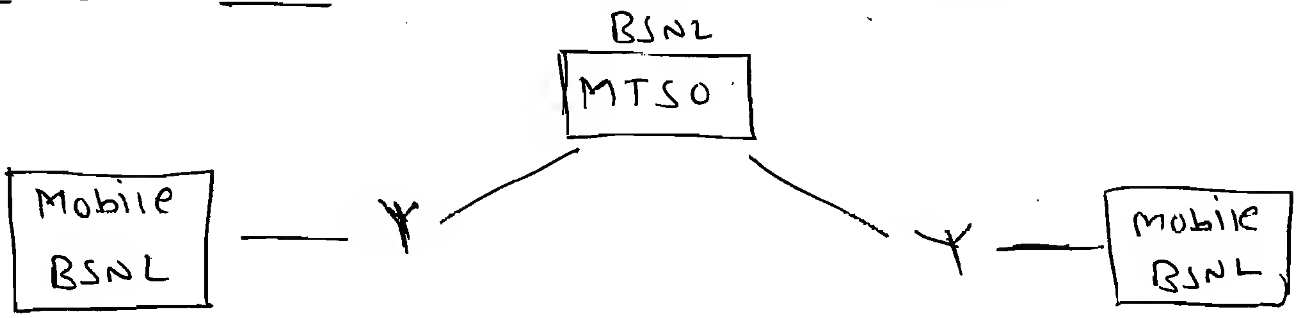
⇒ There are three case :

⊙ Case - (i) : BSNL Landline & BSNL mobile Phone.

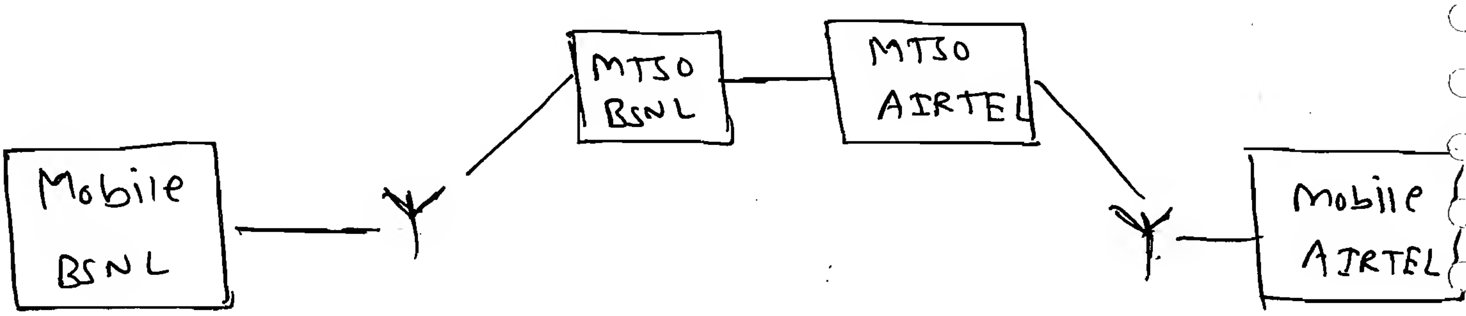
⇒



⊙ Case - ii : BSNL mobile phones or both users.



⊙ Case - iii : BSNL mobile phone & AIRTEL mobile phone.



*

1 G ⟶ Analog ⟶ FM

2 G ⟶ Digital ⟶ QPSK } R_b = KbPS.

2.5 G ⟶ Digital ⟶ GMSK }

(Gaussian minimum shift Keying).

3 G ⟶ Digital ⟶ mbps video.

4 G ⟶ 100 mbps HD video.